UTE OF SCIENC

The Directly-Useful **D.U.** Technical Series

Founded by the late Wilfrid J. Lineham, B.Sc., M.Inst.C.E.
General Editor: John L. Bale.

Line Charts for Engineers

BY

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FOUNDER'S NOTE

THE DIRECTLY USEFUL TECHNICAL SERIES requires a few words by way of introduction. Technical books of the past have arranged themselves largely under two sections: the Theoretical and the Practical. Theoretical books have been written more for the training of college students than for the supply of information to men in practice, and have been greatly filled with problems of an academic character. Practical books have often sought the other extreme, omitting the scientific basis upon which all good practice is built, whether discernible or not. The present series is intended to occupy a midway position. The information, the problems, and the exercises are to be of a directly useful character, but must at the same time be wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind. We shall thus appeal to all technical people throughout the land, either students or those in actual practice.

AUTHOR'S PREFACE

It is possible to place too high a value on mechanical aids to calculation, for slavish reliance on them must tend to the lessening of the ability to compute, and to the loss of confidence in one's working. Judiciously chosen, however, and used with reason, many of these aids prove to be of great service, and amongst them may be classed the various forms of charts.

Interest in the study of nomography, since its presentation by *D'Ocagne*, has been stimulated by the writings of *Hezlet*, *Peddle*, *Lipka*, *Brodetsky* and others: but it would seem that the practice of the principles is restricted to comparatively few of those whom it might benefit. It is felt, therefore, that there is a field of usefulness for any book which may assist towards a proper appreciation of the subject.

No book can include a chart for every type of equation: all that can be done is to indicate on broad lines how to adapt a formula for representation by a chart, and to show how the chart may be most easily built up. The object of this present work is primarily the latter; the greatest possible attention being given to the details of the construction: and for the sake of clearness the charts are not crowded with markings, only sufficient being shown to indicate the nature of the graduation of the axes.

The knowledge of mathematics required for the understanding of these charts is small, and it is suggested that the matter is so arranged and described that anyone previously unacquainted with the subject of charts can follow with ease every case treated.

Stress is laid at the commencement on the very essential groundwork of "functional scales," the notion of a "scale-value" of 1 inch representing *n* units of a function being adopted in place of the "modulus" of *n* inches per unit, since this seems to tend to simplicity. The different types of charts are introduced progressively, the grading being as nearly as possible according to difficulty. The examples chosen are such as should appeal to the engineer, but the general subject-matter has a wider interest.

The Author's thanks are due to Mr. Clifford Hall for his skilful preparation of the diagrams for the press, and to Mr. J. L. Bale for valuable assistance at all stages of the work.

Notification of any errors remaining will be considered as a favour.

W. N. Rose.

Borough Polytechnic, S.E. 1.
February 1923.

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PREVIOUS KNOWLEDGE

THROUGHOUT the book the fullest explanations of the processes are given consistent with the nature of the work, and the reader having a fair knowledge of the rules of elementary mathematics should have no difficulty in following every step.

The mathematical attainment necessary may be summarised as follows:—

A good working knowledge of the manipulation of logarithms, with particular reference to the rules

$$\log(AB) = \log A + \log B$$
$$\log A^n = n \log A$$
$$\log\left(\frac{A^n}{B^m}\right) = n \log A - m \log B$$

Ability to understand the transposition of the symbols in such an equation as $\frac{a^n b}{c^m} = d$

Thus $a = \sqrt[n]{\frac{dc^m}{b}}$, $b = \frac{dc^m}{a^n}$, and $c = \sqrt[m]{\frac{a^n b}{d}}$

A slight acquaintance with the trigonometric ratios, and with the laws of similar geometrical figures.

LINE CHARTS FOR ENGINEERS

CHAPTER I

INTRODUCTION.

WITH the increasing appreciation of the great usefulness of graphs generally has come also a development of geometrical treatment to which D'Ocagne applied the name "Nomography." The derivation of the word, from the Greek word *nomos*—a law, is indicative of the subject, which concerns the graphic representation of laws in a rather special manner.

In the years that have elapsed since D'Ocagne first published his "Traité de Nomographie" in 1899, great progress has been made in the development of "nomography" and in the application of "nomograms," or "alignment charts," to the solution of technical problems.

We may define a nomogram, or, as it is also called, a "nomograph," as a chart designed to correlate the variables associated in a law by the simple expedient of drawing straight lines on the diagram. The axes in the diagram may be straight or curved, or they may be parallel to one another or inclined at any angle. The name "alignment chart" is usually applied to a chart in which the axes are all vertical: these charts are certainly those most frequently encountered, but they are by no means the only nomograms of value.

Nomograms possess the virtue of simplicity of construction and use: a nomogram expressing a certain law when once constructed can be used with ease by one who lacks all understanding of the significance of the law and of its manipulation. The chart does away with much transformation of formulæ and calculation, and the time and labour spent on the preparation of the chart are soon regained by its employment.

A knowledge of the method of the construction of the chart is not a necessity for the using of the chart, but it may on occasion be a powerful aid: whilst to the designer of charts the principles underlying the construction are essential. Hence there is great need for detailed description of the various forms of chart, guidance concerning the best choice of chart for special conditions, and hints about the actual drawing of the charts.

Whilst the greater part of this work relates to nomograms, the importance of particular forms of intercept charts has not been forgotten, some examples of these being also included.

Functional Scales.—As so much in the construction of alignment charts depends upon the correct appreciation of the term "a functional scale," it is advisable to consider in detail the setting-out of these scales before proceeding to the actual charts. A functional scale is one in which the function of a variable is represented along one axis only, the marking placed at any point being the value of the variable at that point, and not the value of the function. By setting out the scale along one line a great saving of space is effected. Thus we might represent the function x^2 by the ordinary parabola obtained by the plotting of $y = x^2$, in which case two graduated axes and a curve would be necessary; but it will be shown that this function can quite as easily be represented along one graduated axis only.

The whole difficulty in the construction of a functional scale rests in the selection and use of what may be called the "scale-value," and the variety of types here discussed should assist towards the removal of that difficulty.

In place of the familiar x and y, associated with co-ordinate axes, we shall use the letters u and v to denote the variables, and in these cases of functional scales we shall need u only.

The very simplest type of functional scale is that which represents the variable u itself: the scale chosen will evidently be of the nature $1'' = n$ units of u, such a scale being termed a "natural" scale, as distinct from a "logarithmic" scale.

Two forms of scale for the representation of the simple function u are shown in Fig. 1, the two scales differing in the fact that the scale-values are not alike. In (*a*) 1 inch represents 1 unit of u and in (*b*) 1 inch represents 5 units of u; and if it be agreed to define the scale-value (denoted by l) as "the number of units per inch," then in (*a*) $l = 1$ and in (*b*) $l = 5$.

These two scales are set out by marking from an ordinary evenly-divided scale, and it will be seen that in each case the marking for a value u_1 is at an actual distance $\frac{u_1}{l}$ from the zero of the scale. Thus the mark for 3 is put at a distance of $\frac{3}{1}$ inches in (*a*), and $\frac{3}{5}$ inch in (*b*), from the mark for 0.

To set out a scale of u^2, the procedure is as follows:—Supposing that the highest value of u to be taken is 6, we observe that $6^2 = 36$ and that a convenient length for this number is 3·6 inches. Thus 3·6 inches represent 36 units of u^2 or 1″ represents 10 units, so that $l = 10$; also the actual distance from the zero marking of a value u_1 must be $\frac{u_1^2}{l}$ or $\cdot 1 u_1^2$. The marking for $u = 1$ is distant $\cdot 1 \times 1^2$, *i. e.* ·1 inch from 0; that for 2 is distant $\cdot 1 \times 2^2$, *i. e.* ·4 inch from 0, and so on; the graduated axis being shown at (*c*), Fig. 1.

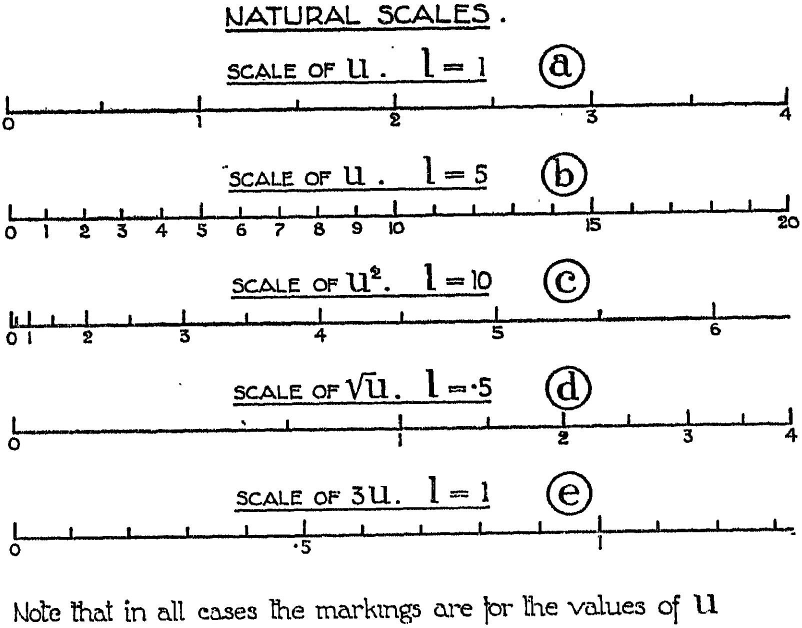


FIG. 1.—Functional Scales.

It must be clearly understood that the markings in each case are those for u, and not for the function of u.

It would be possible to set out an evenly divided scale for u^2, marking the values as those for u^2 and not u, but if this were done in a chart it would be necessary, in order to find the value of u, to extract the square root of the value read off, so that the chart would lose much of its value.

At (*d*), Fig. 1, is shown a scale of $u^{\frac{1}{2}}$ or $\sqrt{u}$, the value of l being taken as ·5, *i. e.* 1″ = ·5 unit of $\sqrt{u}$. Then the marking for 2, for example, is placed at a distance of $\frac{\sqrt{2}}{\cdot 5}$, or 2·83 inches from the zero of the scale.

An axis graduated to show values of $3u$ is shown at (*c*), Fig. 1, the value of l being taken as 1. The marking for $u = \frac{2}{3}$ is placed at a distance from the zero of $\frac{3 \times \frac{2}{3}}{1}$, *i. e.* 2 inches, and so on.

Natural scales, although easy to set out, are of limited value only, and for the majority of charts logarithmic scales are necessary. The graduation of these is made according to the same plan as adopted for the natural scales, *i. e.* the actual distance in inches from the zero marking (which is 1 for a logarithmic scale, since $\log 1 = 0$) is always given by the value of $\frac{f(u)}{l}$.

The slide-rule scales furnish good examples of logarithmic scales, and, in fact, they may often be used with great advantage in the construction of charts. The lengths of the graduated portions of the rules vary slightly with the different makes of rule, but an average value of the length from 1 to 100 on the A scale is 25 cm., or 9·84 inches. Thus if an axis be graduated, the A scale of the slide rule being used, the value of l is $\frac{1}{4 \cdot 92}$ or ·2033; for 9·84 inches is the length from log 1 to log 100, *i. e.* from 0 to 2, so that 9·84 inches represent 2 units, and hence 1 inch represents $\frac{2}{9 \cdot 84}$, *i. e.* ·2033 unit.

To set out a scale of $\log u$ without using a scale divided logarithmically, it is necessary to determine first the length of axis required and from that to fix a suitable scale-value. Thus suppose it is required to graduate an axis for $\log u$ to show values of u from 1 to 100, the length of line not to exceed 4″, the scale-value might conveniently be ·5, since, in that case, 4″ would represent the difference between log 1 and log 100. Hence $l = \cdot 5$ and the distance from the marking for 1 to any marking u_1 is $\frac{\log u_1}{\cdot 5}$ or $2 \log u_1$; thus the marking for 3 is placed at a distance of $2 \times \cdot 4771$, *i. e.* ·9542 inch from the marking for 1, and the marking for 60 is distant $2 \times 1 \cdot 7782$, *i. e.* 3·556 inches from the marking for 1. This scale is shown at (*a*), Fig. 2.

Functions of u involving awkward powers or roots are in practically all cases treated logarithmically; it is not impossible, however, to set out a natural scale for such a function as $u^{\cdot 85}$, but it is decidedly more convenient to make the scale a logarithmic one. Suppose it is desired to set out a scale for $\log u^{\cdot 85}$, the values of u ranging from 10 to 250: then, since $\log u^{\cdot 85} = \cdot 85 \log u$, the length of the axis must be $\cdot 85(\log 250 - \log 10)$, *i. e.* $\cdot 85(2\cdot 3979 - 1) = 1\cdot 188$ units. Then if 3″ represents 1·2 units, 1″ represents ·4 unit and $l = \cdot 4$, so that the distance from the marking for 10 (in inches), of a marking $u_1 = \dfrac{\cdot 85(\log u_1 - \log 10)}{\cdot 4}$, *i. e.* $2\cdot 125(\log u_1 - 1)$. Thus $u = 100$ is placed at a distance of $2\cdot 125(2 - 1)$, *i. e.* 2·125 inches from $u = 10$; and $u = 190$ is distant $2\cdot 125(2\cdot 2788 - 1) = 2\cdot 717$ inches from $u = 10$. This scale is shown at (*b*), Fig. 2. In all cases in which distances have to be calculated it is desirable to set out the calculations in a table, just as would be done for the plotting of an ordinary graph.

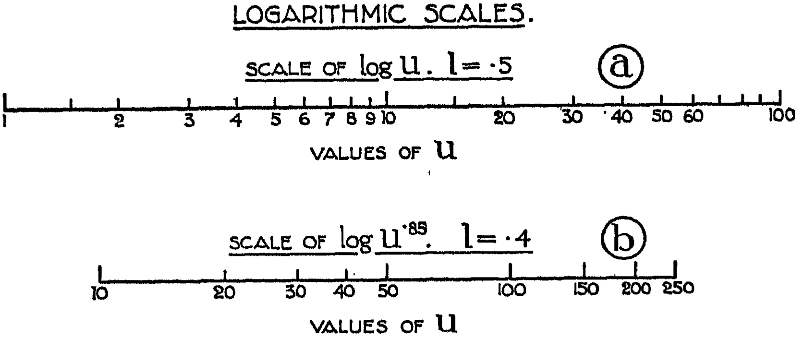


FIG. 2.—Functional Scales.

To illustrate further the construction of these functional scales two other examples are added, the one requiring a natural scale and the other a logarithmic scale.

Example 1.—Set out a scale of ·005D, the values of D ranging from 3 to 10.

If D ranges from 3 to 10, ·005D ranges from ·005 × 3 to ·005 × 10, *i. e.* its range is ·035. Let 3·5 inches represent this range so that 1 inch represents $\dfrac{\cdot 035}{3\cdot 5}$ unit of $f(D)$, *i. e.* $l = \cdot 01$.

Then the actual distance from the virtual zero (in this case the marking for 3) of a marking $D_1 = \dfrac{\cdot 005(D_1 - 3)}{\cdot 01}$ or $\cdot 5(D_1 - 3)$, so that the marking for 7 is placed at a distance of $\cdot 5(7 - 3)$, *i. e.* 2″ from the marking for 3. The scale, being a natural scale, presents no difficulty; it is shown at (*a*), Fig. 3.

Example 2.—Graduate an axis to give values of $\log v^{1\cdot4}$, v ranging from 4 to 14.

$\operatorname{Log} v^{1\cdot4} = 1\cdot4 \log v$ and the total length of the axis must be $1\cdot4(\log 14 - \log 4) = 1\cdot4(1\cdot1461 - \cdot6021) = \cdot7616$ unit. If 3·81 inches be taken to represent this range, 1 inch represents ·2 unit and $l = \cdot2$.

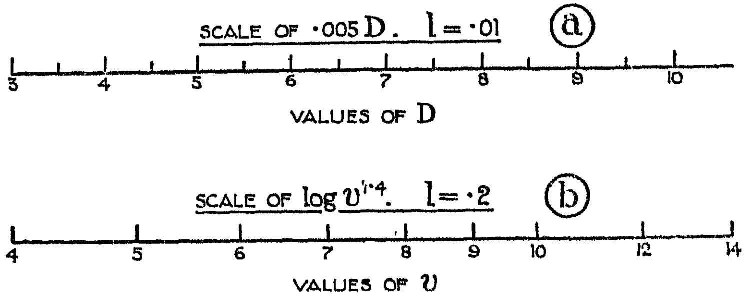


FIG. 3.—Functional Scales.

Then the distance from the virtual zero (viz. the marking for $v = 4$) of the marking for $v_1 = \frac{1}{\cdot2} \times 1\cdot4(\log v_1 - \log 4)$

$$= 7(\log v_1 - \cdot6021).$$

The table for the graduation reads

v	4	5	6	7	8	9	10	12	14
log v	·6021	·6990	·7782	·8451	·9031	·9542	1	1·0792	1·1461
log v − log 4	0	·0969	·1761	·2430	·3010	·3521	·3979	·4771	·5440
7 × difference of logs = distance (inches) .	0	·6783	1·2327	1·701	2·107	2·4647	2·7853	3·3397	3·808

The plotting is shown at (*b*), Fig. 3.

Scales representing the trigonometric functions are valuable, and hence two examples are included here.

Example 3.—Graduate an axis to represent the function 3 sin 2A, the angle A varying from 0° to 90°.

Although the actual range of A is 90°, the effective range is 45°, for if A = 45°, 2A is 90°, and sin 2A has then its greatest value. Now $\sin 0° = 0$ and $\sin 90° = 1$, so that the axis must be sufficiently long to show 3×1 units. If 3 inches represent 3 units, then the scale-value is 1. The distance from the zero to a marking for A_1 is thus $\frac{3 \sin 2A_1}{1}$ inches, and a table of distances can be compiled, use being made of the rule $\sin \alpha = \sin (180 - \alpha)$ when A exceeds 45°; a few values being

A	0	10°	20°	30°	40°	45°	50°	60°	70°	80°	90°
2A	0	20°	40°	60°	80°	90°	100°	120°	140°	160°	180°
sin 2A	0	·342	·6428	·866	·9848	1	·9848	·866	·6428	·342	0
3 sin 2A = distance from zero marking	0	1·026	1·9284	2·598	2·9544	3	2·9544	2·598	1·9284	1·026	0

The graduated axis is shown at (*a*), Fig. 4.

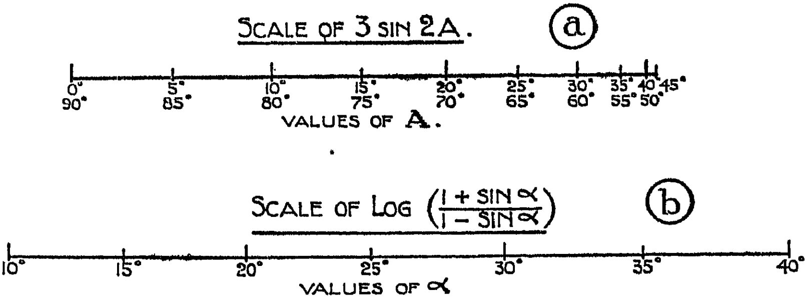


FIG. 4.

Example 4.—Graduate an axis to represent the function $\log\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)$, the range of α being from 10° to 40°.

When $\alpha = 10^\circ \quad \sin\alpha = \cdot 1736$

$$\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{1\cdot 1736}{\cdot 8264} \text{ and } \log\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right) = \cdot 1523$$

and when $\alpha = 40^\circ \quad \sin\alpha = \cdot 6428$

$$\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{1\cdot 6428}{\cdot 3572} \text{ and } \log\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right) = \cdot 6626$$

so that a range of ·6626 — ·1523 unit must be shown, a suitable scale being 1 cm. = ·05 unit. Then the table of values is:—

α	10°	15°	20°	25°	30°	35°	40°
$\sin\alpha$	·1736	·2588	·342	·4226	·5	·5736	·6428
$\log\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)$	·1523	·2302	·3096	·3917	·4771	·5672	·6626
Difference of logs	0	·0779	·1573	·2394	·3248	·4149	·5103
Distance in cms. from virtual zero	0	1·558	3·146	4·788	6·496	8·298	10·206

the figures in the fourth line being obtained by subtracting ·1523 from each log in the third line; and the fifth line being obtained from the fourth line by dividing by ·05. The graduated axis is shown at (*b*), Fig. 4.

It is sometimes desirable to graduate an axis on its two sides so that two variables may be related. Thus in Fig. 5 the axis is graduated with a logarithmic scale on one side and a natural scale on the other. The relation between the variables is given by the equation

$$\frac{T}{t} = e^{\mu\theta}$$

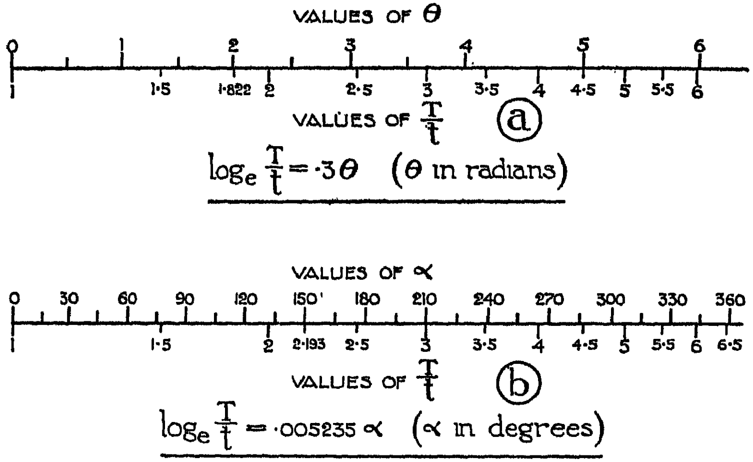


Fig. 5.—Tension in a Belt.

T and t being the tensions at the ends of a belt passing round a pulley, μ being the coefficient of friction between belt and pulley, and θ being the angle of lap in radians. For the case illustrated, μ is taken as ·3 and θ ranges from 0 to 6·283, *i. e.* from 0 to 360°.

If
$$\frac{T}{t} = e^{\mu\theta}$$
then
$$\log_e \frac{T}{t} = \mu\theta.$$

Now $\mu = \cdot 3$, and thus $\mu\theta$ ranges from 0 to $\cdot 3 \times 6{\cdot}283$, *i.e.* from 0 to 1·8849; and a convenient scale-value for the representation of this is $l_\theta = \cdot 5$ unit. The axis may now be graduated, it being remembered that the markings are for values of θ, the distances from the zero being of the nature $\frac{\cdot 3\theta}{\cdot 5}$, *i. e.* ·6$\theta$, and the scale being a natural scale. The range of $\log_e \frac{T}{t}$ is also 1·8849, and so the same scale-value can be chosen, viz. $l_2 = \cdot 5$, but this second scale is a logarithmic scale, and to avoid any confusion it is best to make up a table of distances, thus :—

$\frac{T}{t}$	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6	6·5
$\log_e \frac{T}{t}$ . . .	0	·4055	·6931	·9163	1·0986	1·2528	1·3863	1·5041	1·6094	1·7048	1·7918	1·8718
$2 \log_e \frac{T}{t}$ = distance from marking for 1	0	·811	1·3862	1·8326	2·1972	2·5056	2·7726	3·0082	3·2188	3·4096	3·5836	3·7436

It will be seen from (*a*), Fig. 5, that if the angle of lap is 2 radians the value of $\frac{T}{t}$ level with 2 on the scale of θ is read off as 1·822.

It is probable that in most cases the angle would actually be measured in degrees. If α is the angle in degrees the formula becomes

$$\log_e \frac{T}{t} = \frac{\mu\alpha}{57{\cdot}3} = \frac{{\cdot}3 \times \alpha}{57{\cdot}3} = {\cdot}005235\alpha.$$

The range is the same as before, viz. 0 to 1·8849, so that the same scale-value may be taken, viz. $l_1 = \cdot 5$. Then the distance of a marking for α_1 from the zero $= \frac{\cdot 005235\alpha_1}{\cdot 5} = \cdot 01047\alpha_1$ inch and the scale can be graduated accordingly, as at (*b*), Fig. 5. For any angle of lap the value of $\frac{T}{t}$ can be read off directly : thus if $\alpha = 150°$, $\frac{T}{t} = 2{\cdot}193$.

If desired, this diagram may be used to change from degrees to radians, or *vice-versa*, by simply drawing a vertical line to cross the axes of θ and α.

CHAPTER II

CHARTS WITH STRAIGHT LINE AXES CARRYING NATURAL SCALES.

THE method of graduating an axis to suit varied requirements having been treated, it is now possible to proceed with the discussion of the process of aligning axes relative to one another so that they may be put to some practical use. It is not easy to classify alignment charts, but perhaps the simplest mode of division is into two main classes, viz. those in the one class having "natural" scales, and those in the other class having "logarithmic" scales, but this method does not allow for the inclusion of charts in which natural scales are used for some axes and logarithmic scales for others. Even with this division there is no great distinction between the two classes, since many of the rules deduced for the former class can with slight modification be adapted for use with the latter class.

Whilst not attempting to apply a rigid classification, therefore, the plan adopted will be to treat the simple charts, with natural scales, before introducing logarithmic scales.

We may commence the discussion by the consideration of a very simple example :—

Let AC and BD (Fig. 6) be two verticals, along which are to be represented values of the two variables, u and v respectively.

Let $1'' = l_1$ units of u along the axis AC
and $1'' = l_2$,, v ,, ,, BD.

It being permissible to place the zero of the scale anywhere along the axis, we may say that AB is to serve as the line of zero values.

Draw a third axis, EF, between AC and BD and so placed that

$$\frac{\text{AE}}{\text{EB}} = \frac{l_2}{l_1}; \text{ and let AE} = m_1, \text{ EB} = m_2 \text{ and AB} = m.$$

Then, if AC represents some value of u, say u_1, and BD represents some value of v, say v_1, we may imagine that EF represents a value w_1 of another variable w which is connected in some way with u_1 and v_1: also, if it be supposed that EF represents w_1 to a scale of $1'' = l_3$ units, where $l_3 = l_1 + l_2$, then we can proceed by the aid of simple geometry to find the connection between w_1, u_1, and v_1, and thus that between w, u, and v.

Draw GHD parallel to AB: then from the similarity of the triangles CGD and FHD

$$\frac{CG}{FH} = \frac{GD}{HD} = \frac{AB}{EB} = \frac{m}{m_2}$$

$$i.e. \quad \frac{AC - AG}{EF - EH} = \frac{m}{m_2} \text{ or } \frac{AC - BD}{EF - BD} = \frac{m}{m_2} \quad . \quad . \quad (1)$$

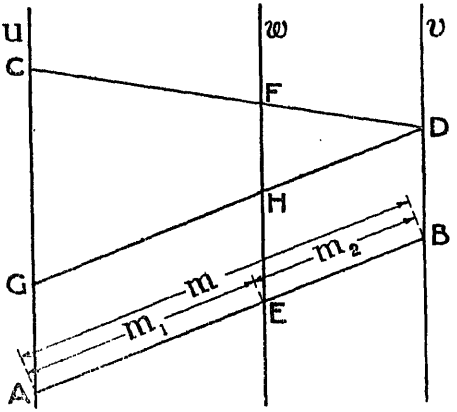


FIG. 6.

Now
$$\frac{m_1}{m_2} = \frac{l_2}{l_1} \text{ and thus } \frac{m_1}{m_2} + 1 = \frac{l_2}{l_1} + 1$$

$$\frac{m_1 + m_2}{m_2} = \frac{l_2 + l_1}{l_1}$$

$$\frac{m}{m_2} = \frac{l_2 + l_1}{l_1} = \frac{l_3}{l_1}.$$

Also AC is an actual length, representing u_1 to a scale of l_1 units per inch, so that AC × $l_1 = u_1$ and AC $= \frac{u_1}{l_1}$, and in like manner

$$BD = \frac{v_1}{l_2} \text{ and } EF = \frac{w_1}{l_3}$$

Then, by substitution in equation (1)

$$\frac{\dfrac{u_1}{l_1}-\dfrac{v_1}{l_2}}{\dfrac{w_1}{l_3}-\dfrac{v_1}{l_2}}=\frac{l_3}{l_1}$$

whence
$$\frac{l_2u_1-l_1v_1}{l_2w_1-l_3v_1}=1$$

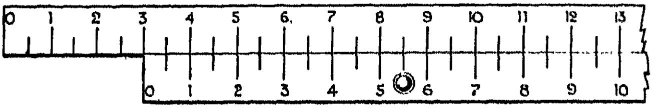
$$l_2w_1-l_3v_1=l_2u_1-l_1v_1$$

$$l_2w_1=l_2u_1-l_1v_1+l_1v_1+l_2v_1=l_2(u_1+v_1)$$

and
$$w_1=u_1+v_1.$$

It has thus been proved that the intercepts on the three vertical axes can be used to connect the three variables, the relation between them in this case being $w=u+v$.

By this placing of the mid-vertical in its correct position relative to the fixed axes, the process of addition can be performed mechanically, the space required being much less than for an arrangement of sliding scales as illustrated in Fig. 7.



SLIDING SCALES FOR ADDITION

Fig. 7.

From a chart for addition to one for subtraction is a small step; and it can now be demonstrated that subtraction can be performed in a somewhat similar manner, provided that either (*a*) the u and v scales are graduated in opposite directions, or (*b*) the "mid-vertical" is placed outside the axes of u and v. In case (*a*), lettering the diagram (Fig. 8) as for Fig. 6, we have the readings for u upward and those for v downward.

Then, as before,
$$\frac{\mathrm{CG}}{\mathrm{FH}}=\frac{m}{m_2}$$

whence
$$\frac{\dfrac{u_1}{l_1}+\dfrac{v_1}{l_2}}{\dfrac{w_1}{l_3}+\dfrac{v_1}{l_2}}=\frac{m}{m_2}=\frac{l_3}{l_1}$$

$$\frac{l_2u_1+l_1v_1}{l_2w_1+l_3v_1}=1$$

and
$$w_1=u_1-v_1.$$

In case (*b*) let the diagram be as in Fig. 9, the axis of w being

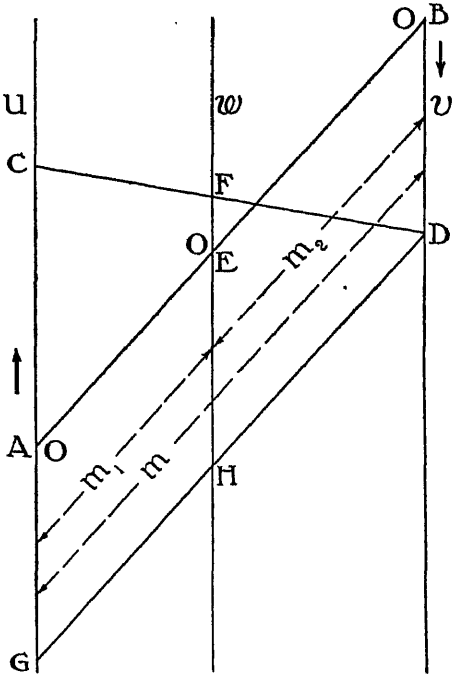


FIG. 8.

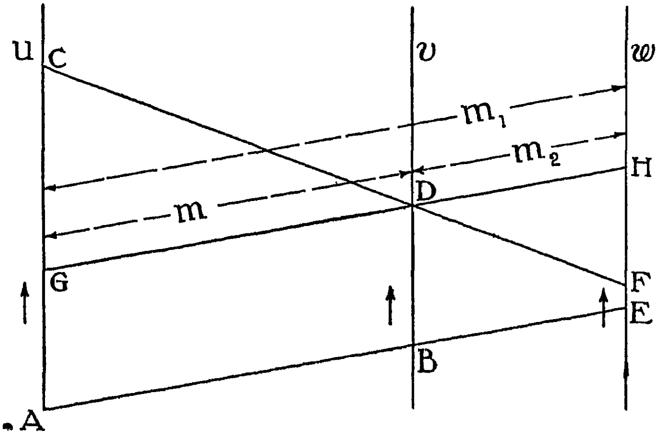


FIG. 9.

taken to the right of the axis of v, and being so situated that $\frac{m_1}{m_2} = \frac{l_2}{l_1}$: also let $l_3 = l_2 - l_1$.

By similar triangles $\frac{CG}{FH} = \frac{m}{m_2}$

$$\frac{\frac{u_1}{l_1} - \frac{v_1}{l_2}}{\frac{v_1}{l_2} - \frac{w_1}{l_3}} = \frac{m}{m_2} = \frac{l_3}{l_1}$$

$$\frac{l_2u_1 - l_1v_1}{l_3v_1 - l_2w_1} = 1$$

$$l_2w_1 = l_2v_1 - l_1v_1 - l_2u_1 + l_1v_1$$

or $$w_1 = v_1 - u_1.$$

It should be evident that the chart for addition, shown in Fig. 6, could also be used for subtraction, and thus Fig. 9 is in reality a slight modification of Fig. 6.

Chart for Addition

Example 1.—Construct a chart to give values of w, u, and v in the equation $w = u + v$, the values of u ranging from 0 to 10, and v varying from 20 to 70.

[The ranges of the values have been thus selected so that it can be made quite clear that the line of zero values is not an absolute necessity; the line joining 0 on the axis of u to 30 on the axis of v takes its place in this example.]

To Construct the Chart.—Draw two vertical lines, AC and BD (Fig. 10), 3″ apart, so that $m = 3$. Along AC, which is to be the axis of u, set out the values of u to a scale of 1″ = 2 units: thus $l_1 = 2$. Select some convenient point on BD, such as F, and indicate this as the virtual zero of the v scale, actually placing the marking 30 there. Set out from F along BD the scale of v, taking the value of l_2 as 10. Then MH, the axis of w, must be so placed that

$$\frac{AM}{AB} = \frac{l_2}{l_1 + l_2} = \frac{10}{2 + 10}$$

or $$AM = 2\tfrac{1}{2}'',$$

and the scale-value for the axis of w is given by $l_3 = l_1 + l_2 = 12$.

To fix a starting point on this axis, join 0 on the axis of u to 30 on the axis of v, giving the line AGF which cuts the axis MH at G, and mark G as 30 on the scale of w; this being permissible since $w = u + v$, and if $u = 0$ and $v = 30$, w must also be 30. Complete the chart by graduating the axis of w to the scale of 1″ = 12 units, using a scale divided into twelfths of an inch.

To Use the Chart.—Given $u = 6$ and $v = 52$, to find w. Join 6 on AC to 52 on BD and note the point of intersection of this line with the axis MH, viz. 58.

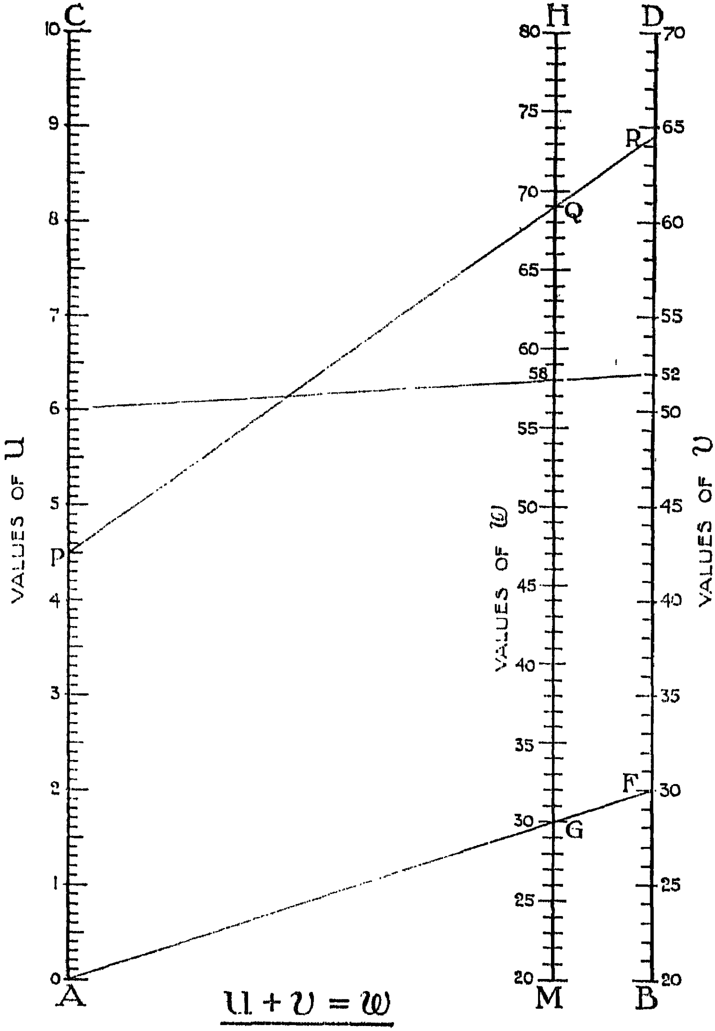


FIG. 10.—Chart for Addition.

Given $u = 4{\cdot}5$ and $w = 69$, to find v, proceed as indicated by the line PQR, the value 64·5 being read off at the point R.

To obviate the obliteration of the charts by the drawing of numerous lines upon them, it is desirable to draw a line on a piece of tracing-cloth, or, better still, upon a strip of celluloid. This can be used over and over again, and can be moved up and down on the diagram, thus saving time and preserving the chart.

The same method of construction would be employed for a chart to represent the more general equation $au + bv = w$,

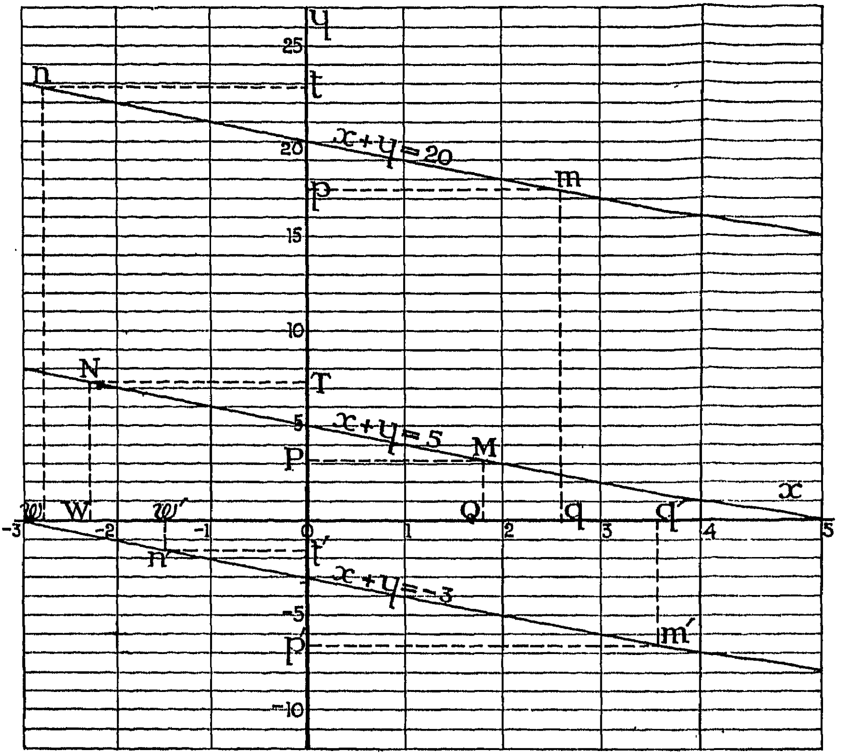


FIG. 11.

but it must be remembered that the scale value l_1 would be determined from the range in the values of au, although the axis would be marked with the values of u.

The Derivation of a Nomogram from a Graph.—The nomogram with parallel axes representing the equation $u + v = w$ can be obtained from the graphs, with Cartesian co-ordinates, representing the equation $x + y = c$, where c has any value, in the following simple manner:—

Let MN (Fig. 11) be the line $x + y = 5$. Selecting any point M upon it, note the co-ordinates MP and MQ the two parallel axes AR and BK (Fig. 12) 3 inche making AB the line of zero values. From A set off AR thus setting off an x value along the axis of u, and fro make BS = MQ: also join RS. Repeat the process for s other point N, making AH = NT and BK = NW. The l RS and HK intersect at G, and it can be shown that all lines

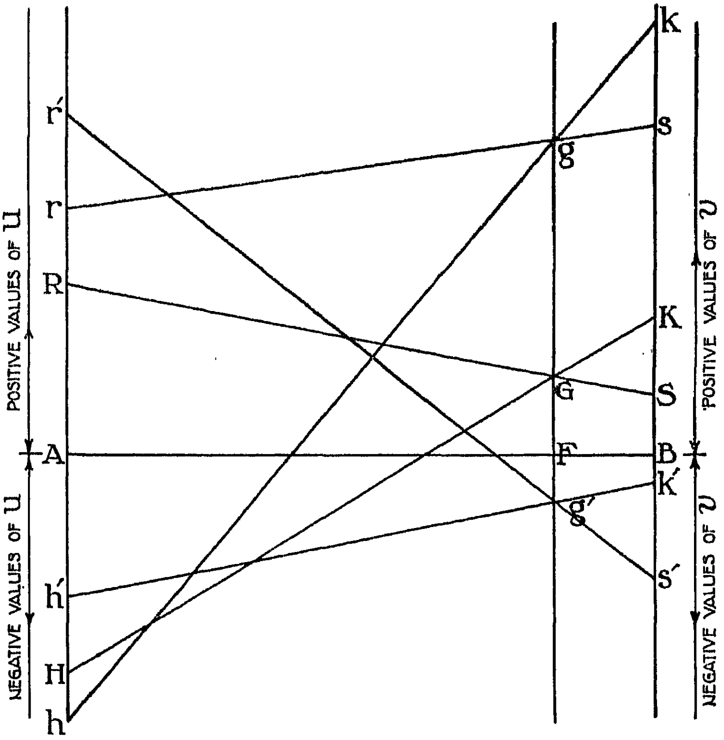


FIG. 12.

like RS and HK, determined in the same way from other points on the line MN, must pass through G.

Now let mn (Fig. 11) be the line $x + y = 20$ and deal with the points m and n as before described: thus $Ar = mp$, $Bs = mq$, $Ah = nt$ and $Bk = nw$. It is seen that the intersection g of the lines hk and rs lies vertically above G, the distance Fg being exactly four times FG, *i. e.* $\frac{Fg}{FG} = \frac{4}{1}$ or $\frac{20}{5}$.

Treating the line $m'n'$ (Fig. 11), *i. e.* the line $x + y = -3$, in the same way, the point of intersection of $r's'$ and $h'k'$ is found to be below AB, but on the vertical through G and g, and the distance Fg' is seen to be $\frac{3}{5}$ of FG. Hence if the axis Fg was graduated from F as zero, markings could be placed at g, G, and g', 20, 5, and -3 respectively. These are the values of c in the equations for the particular line from which the respective points g, G, and g' were obtained, and hence it can be seen that the nomogram could be obtained directly from the graph by transferring co-ordinates to the proper axes and finding the locus of the points of intersection of the resulting lines.

In practice, it is much easier to work according to the plan outlined in Example 1, p. 14, but the connection between the two forms of diagram should be noted.

Actually in this particular case the scale-values are $l_1 = 2$ and $l_2 = 10$, and hence we should expect the axis of w to be $2\frac{1}{2}$ inches from that of u, and l_3 to be 12, giving the lengths Fg, FG, and Fg' as $\frac{20}{12}$, $\frac{5}{12}$, and $\frac{3}{12}$ inches respectively. Measurements on the diagram (Fig. 12) confirm these suppositions.

Example 1 is of a most elementary character, in that evenly divided scales alone are required. The method can, however, be modified to suit a case of far greater difficulty, and one in which the advantage of the chart is more markedly demonstrated.

Example 2.—Construct a chart to give values of u, v, and w when connected by the equation $2u + v^3 = w^2$, the range of values of u being from 0 to 20, and that of v being from 0 to 3·5.

The equation $2u + v^3 = w^2$ can be written as $U + V = W$ where $U = 2u$, $V = v^3$ and $W = w^2$, and is therefore of the type illustrated in Example 1, p. 14.

To select the scales for the axes of u and v, it should be noted that if u ranges from 0 to 20, $2u$ varies from 0 to 40 : then if 4″ be taken as the length for this range, 1″ must represent 10 units, *i. e.* $l_1 = 10$. Also, since v varies from 0 to 3·5, v^3 varies from 0 to 42·875 and thus l_2 can also be taken as 10. Then $l_3 = l_1 + l_2 = 20$ and $\frac{m_1}{m_2} = \frac{l_2}{l_1} = 1$.

To Construct the Chart.—Draw the axes of u and v (Fig. 13) 3″ apart and the axis of w mid-way between them. Along the axis of u set out the values of u to the scale of 1″ = 10 units of $2u$, or 1″ = 5 units of u. Along the axis of v set out the values of v to a scale of 1″ = 10 units of v^3. (Thus the marking for $v = 3$ is distant $\frac{3^3}{10}$, *i. e.* 2·7 inches from the zero.) Join the zero on the u scale to the zero on the v scale, and mark the point of intersection with the axis of w as 0, since $w = 0$ when $u = 0$ and $v = 0$. Graduate the axis of w with the scale-value l_3 as 20, *i. e.* place the marking for 8, for example, at a distance from the zero marking of $\frac{8^2}{20}$, *i. e.*

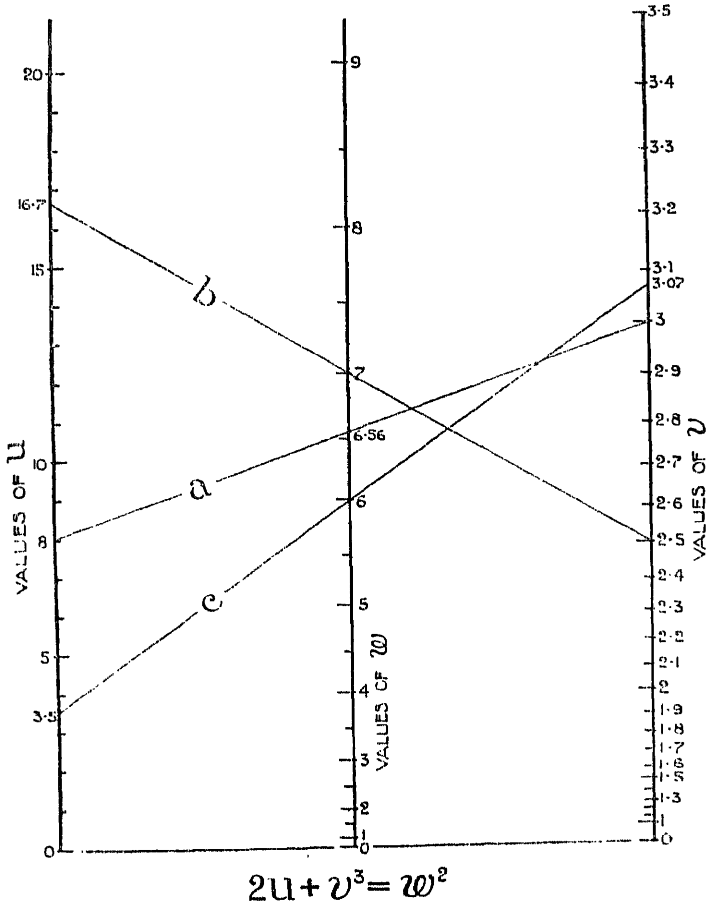


FIG. 13.

3·2 inches. Along the u axis the markings are evenly spaced, since the plotting is of the first power of u only, but along the other axes the distances between the markings for consecutive numbers vary.

To Use the Chart.—(*a*) To find w when $u = 8$ and $v = 3$, join 8 on the axis of u to 3 on the v axis to intersect the axis of w at 6·56 : then 6·56 is the required value.

Proof :— $2 \times 8 + (3)^3 = 43 = (6{\cdot}56)^2$.

(*b*) Given $v = 2{\cdot}5$ and $w = 7$, to find u, join 2·5 on the v axis to 7 on the w axis and produce the line to cut the axis of u at 16·69.

Proof :— $2u + v^3 = w^2$

$$2u = w^2 - v^3 \quad \text{and } u = \frac{w^2 - v^3}{2}$$

Thus $$u = \frac{49 - 15{\cdot}625}{2} = 16{\cdot}688.$$

(*c*) Given $u = 3{\cdot}5$ and $w = 6$, to find v.

Line (*c*) on the diagram joins 3·5 on the u axis to 6 on the axis of w and is then produced to cut the axis of v at 3·07.

Proof :— $v^3 = w^2 - 2u$

so that $v^3 = 36 - 7 = 29$

whence $v = 3{\cdot}073$.

Chart giving Area of Annulus

Example 3.—To construct a chart to give the area of an annulus, using the formula

$$A = {\cdot}7854(D^2 - d^2)$$

D ranging from 0 to 10 and d from 0 to 7.

As a matter of convenience and ease in the subsequent working, it is advisable to transfer ·7854 to the left-hand side of the equation thus

$$\frac{A}{{\cdot}7854} = D^2 - d^2$$

and it will be at once seen that this equation is of the type

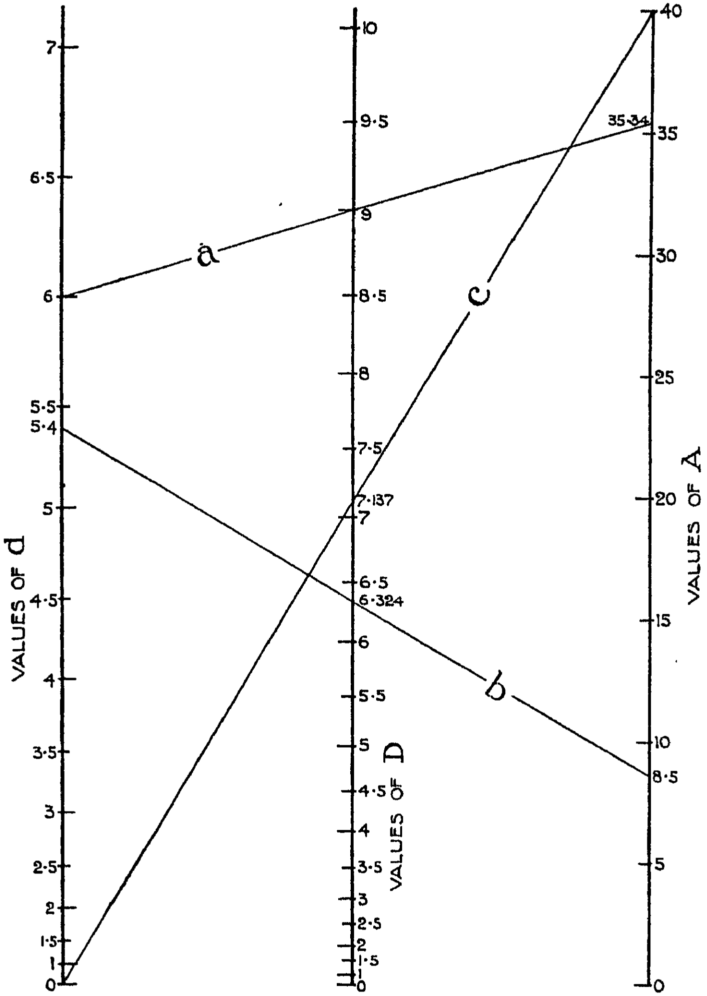
$$w = v - u$$

where $w = \frac{A}{{\cdot}7854}$ $v = D^2$ and $u = d^2$.

Hence the chart may take either the form shown in Fig. 8 or that shown in Fig. 9, the latter form being the more convenient in that a more open scale for the values of the area results, and consequently this form of chart is employed first.

The range of values of d is to be 7, and that of d^2 is therefore 49, so that a scale of 10 units to the inch is suitable; *i. e.* $l_1 = 10$. Also D ranges from 0 to 10 and D^2 from 0 to 100, and hence l_2 may be taken as 20. If $m = 1\frac{1}{2}''$ then $\frac{m_1}{m_2} = \frac{l_2}{l_1} = 2$

and $$m = m_1 - m_2 = 2m_2 - m_2 = m_2$$
so that $$m_2 = 1\tfrac{1}{2}'' \quad \text{and } m_1 = 3''.$$



$$A = \cdot 7854(D^2 - d^2)$$

FIG. 14.—Area of Annulus.

Again, $l_3 = l_2 - l_1 = 20 - 10 = 10$, or $1'' = 10$ units of the function $\dfrac{A}{\cdot 7854}$, and the actual distance in inches from the zero to a marking for $A_1 = \frac{A_1}{\cdot 7854} \div 10 = \frac{A_1}{7 \cdot 854} = \cdot 1274 A_1$. Thus the marking for $A = 20$ is distant $20 \times \cdot 1274$ or 2·55″ from the zero of the A scale.

To Construct the Chart.—Draw the three vertical axes spaced as shown in Fig. 14. Taking some horizontal line as the line of zero values, set out the scales according to the respective values of l_1, l_2, and l_3: thus for $d = 6$ the distance from the zero is $\frac{6^2}{10}$ or 3·6 inches, and for $D = 9$ the distance from the zero is $\frac{9^2}{20}$ or 4·05 inches.

To Use the Chart.—(*a*) To find the area of an annulus of outside dia. 9″ and internal dia. 6″, join 6 on the axis of *d* to 9 on the scale of D and produce the line to cut the axis of A at the required value, viz. 35·34.

(*b*) The area of cross-section of a hollow shaft is 8·5 sq. in., and the internal dia. is 5·4 inches, what is the external dia.?

Join 8·5 on the axis of A to 5·4 on the axis of *d* and note the point of intersection, 6·324, with the D scale: then 6·324 is the outside diameter.

(*c*) What is the dia. of a circle of area 40 sq. in.?

Join 40 on the axis of A to the zero of the *d* scale (since the internal dia. is zero) and note the value of D at the point of intersection with the D axis. This value, viz. 7·137, is the required diameter.

The alternative form of chart for this equation, viz. one similar to that in Fig. 8, is shown in Fig. 15; but it is clearly not so workable as that just discussed. In it the axes of D and *d* are graduated in opposite directions and consequently the axis of A is placed between these. The method of construction should be evident from the chart, the scale-values for which are:—

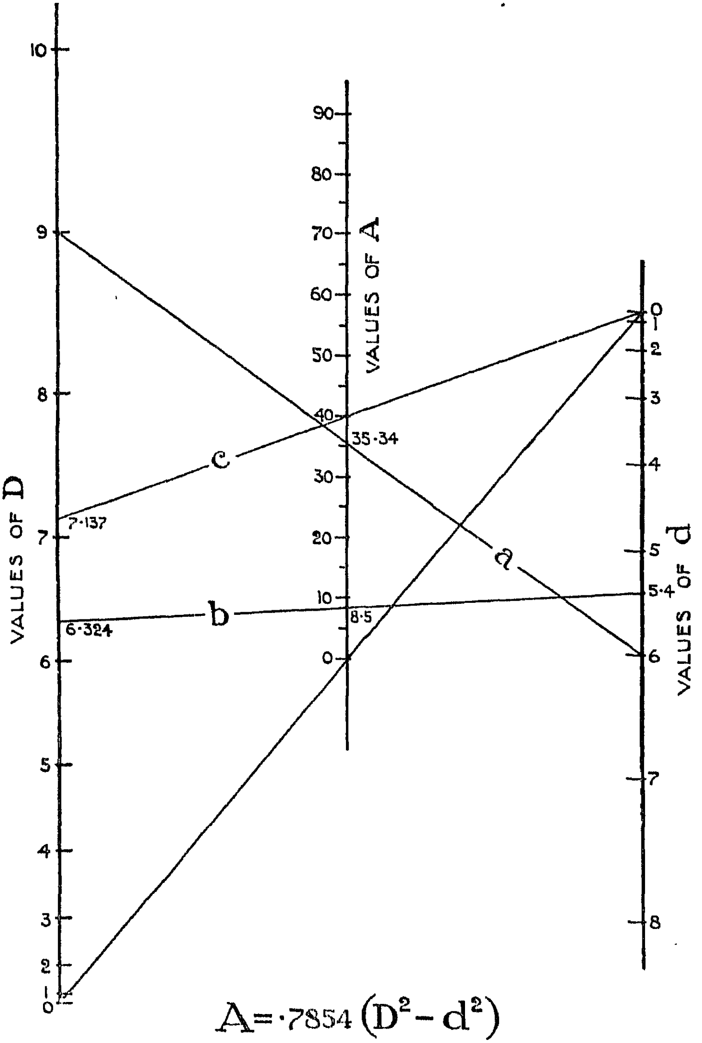
Along the D axis $l_1 = 20$
,, ,, *d* ,, $l_2 = 20$

whence $l_3 = l_1 + l_2 = 40$ units of the function $\frac{A}{\cdot 7854}$

or actually 1″ = 31·42 units of A.

Also $m_1 = m_2 = 1\frac{1}{2}''$: and the lines (*a*), (*b*), and (*c*) indicate the procedure for the three cases detailed above.

A further important formula can be represented by a chart of the form shown in Fig. 14, viz. that connecting the sides of a right-angled triangle, $a^2 + b^2 = c^2$, and the many modifications of it that occur, as, for example, $(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Reactance})^2$ or $(\text{swing radius}_{AB})^2 = (\text{swing radius}_{NN})^2 + (\text{distance between AB and NN})^2$. The advantage



$$A = \cdot 7854\,(D^2 - d^2)$$

FIG. 15.—Area of an Annulus.

of the chart is evident, for the evaluation of this type of expression is often laborious, since logarithms can be used only to a limited extent.

Chart for Multiplication and Division.—A form of chart useful for multiplication or division can be constructed with natural scales only, but the method of setting out the scales is somewhat involved and in general the charts for this purpose have logarithmic scales. An example of this type is, however, included.

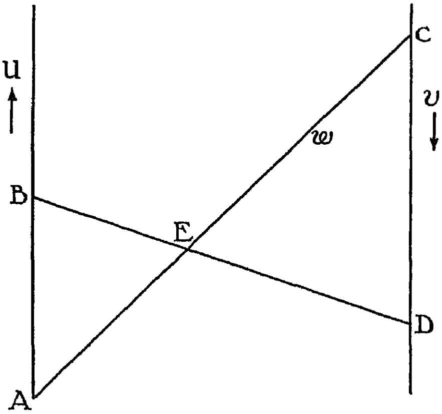


FIG. 16.

In Fig. 16, let AB be the axis of u and let CD be the axis of v, the graduations along this axis being in the opposite direction from those along the axis of u. Then AC, which is any convenient sloping line, connects the zero markings on the two scales, and is termed the axis of w; the relation between the variables being $w = \frac{u}{v}$.

If the scale-values for the u and v axes are l_1 and l_2 respectively, and AB represents some particular value of u, say u_1, and CD represents a particular value v_1, then $\mathrm{AB} = \frac{u_1}{l_1}$ and $\mathrm{CD} = \frac{v_1}{l_2}$.

Let $\mathrm{AC} = m$ and $\mathrm{AE} = m_1$.

Then in the similar triangles ABE and CED

$$\frac{\mathrm{AB}}{\mathrm{AE}} = \frac{\mathrm{CD}}{\mathrm{CE}}$$

so that
$$\frac{u_1}{l_1} \times \frac{1}{m_1} = \frac{v_1}{l_2} \times \frac{1}{m - m_1}$$

$$\frac{u_1}{v_1} = \frac{l_1 m_1}{l_2(m - m_1)}$$

but
$$w = \frac{u}{v} \text{ for all values of } u \text{ and } v.$$

and hence
$$\frac{u_1}{v_1} = w_1$$

Thus $$w_1 = \frac{l_1 m_1}{l_2(m - m_1)}$$

or, solving for m_1, $$w_1 l_2 m - w_1 l_2 m_1 = l_1 m_1$$
$$m_1(l_1 + w_1 l_2) = w_1 l_2 m$$
$$m_1 = \frac{w_1 l_2 m}{l_1 + w_1 l_2}$$

Thus, for any value of w_1 the actual distance m_1 can be calculated, and the marking for w_1 placed at the point so obtained.

A more convenient form of this equation is

$$m_1 = m \times \frac{w_1}{\frac{l_1}{l_2} + w_1}$$

and it will be seen that a uniform scale for w cannot possibly result: consequently the graduation of the axis of w is a tedious process.

Chart for Ohm's Law

Example 4.—Construct a chart to give the values of the variables in the expression of Ohm's law, viz. $C = \frac{E}{R}$, where C is the electric current through a circuit of resistance R, the potential difference being E: the variation of the E.M.F. being from 0 to 200 volts, and the resistance varying from 0 to 8 ohms.

Draw two verticals AD and BF (Fig. 17) 3 inches apart and from A mark off AF = 5″, so that $m = 5$. Along AD set out a scale for values of E, taking 50 units to the inch: thus $l_1 = 50$. From F, but in a downward direction, graduate the axis of R to a scale of 2 units to the inch, so that $l_2 = 2$. Then a few points on the axis of C, *i. e.* the line AF, can be found by joining convenient points on the two axes AD and BF: thus, for example, by joining 200 on the axis of E to 2 on the axis of R, the marking is determined for 100 on the C scale. Further points can be found by calculation, and intermediate points can readily be found by interpolation from a curve.

Thus, since $$\frac{l_1}{l_2} = \frac{50}{2} = 25 \quad \text{and } m = 5$$
$$m_1 = 5 \times \frac{C_1}{25 + C_1}$$

and a table of values reads

C_1 . . .	1	2	5	10	50	100	150	200
$25 + C_1$.	26	27	30	35	75	125	175	225
m_1 . . .	·19	·37	·83	1·43	3·33	4	4·28	4·44

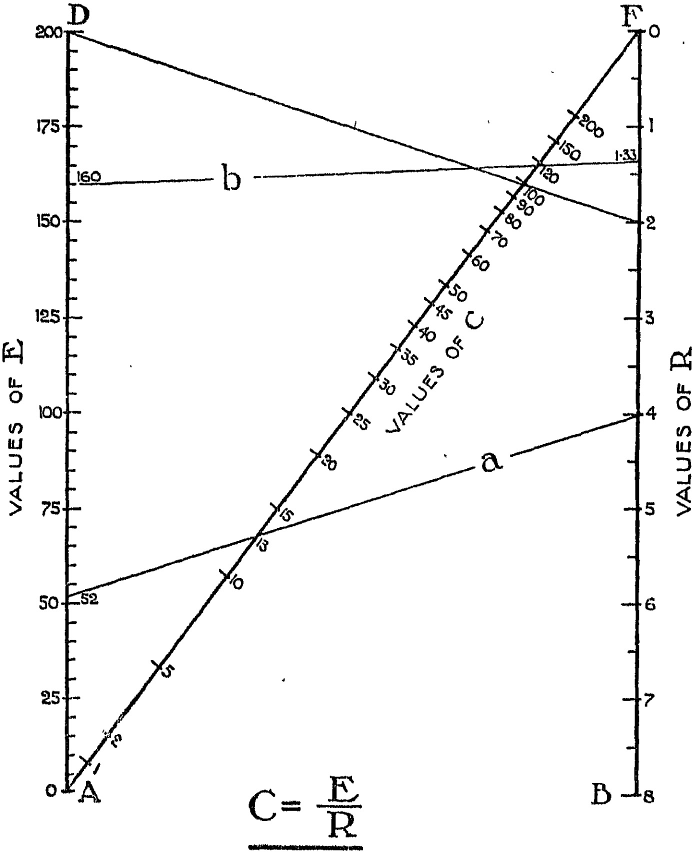


FIG. 17.—Chart for Ohm's Law.

If these values of m_1 are plotted to a base of values of C_1, the scale along the vertical axis being the same as that along the axis AF in Fig. 17, a smooth curve results, from which the distances to be measured along AF can be taken off directly with the dividers. Such a curve is shown in Fig. 18, but the scale of values of m_1 is $\frac{2}{5}$ that of the corresponding scale in Fig. 17.

To Use the Chart.—(*a*) To find C when E = 52 and

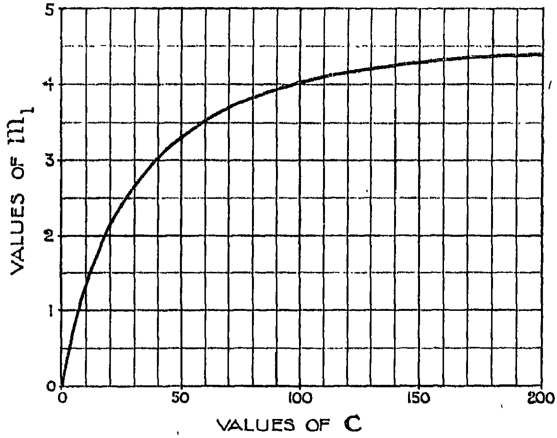


FIG. 18.

R = 4, join 52 on the axis of E to 4 on the axis of R to cut the axis of C at the required value 13.

(*b*) To find R when C = 120 and E = 160, join 160 on the axis of E to 120 on the axis of C and produce the line to cut the axis of R at 1·33.

Proportional Charts.—Essentially all the foregoing might be defined as proportional charts in so far as they depend upon the relations between the sides of similar figures; but charts more apparently "proportional" may be constructed and be peculiarly adapted to certain types of formulæ.

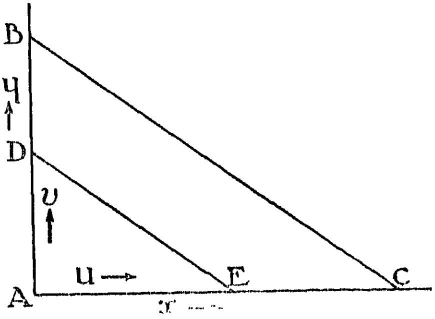


FIG. 19.

Two forms of proportional charts are illustrated in Figs. 19 and 20, that in Fig. 19 being discussed first as the easier to understand.

In Fig. 19 the axis AB is shown perpendicular to the axis AC, but it is not necessary that the angle between the axes should be a right angle if any other is found more convenient : DE and BC are parallel lines; then from the similarity of the triangles ABC and ADE

$$\frac{AB}{AD}=\frac{AC}{AE}$$

Now if AC represents x_1 to a scale of $1''=l_1$ units
AB ,, y_1 ,, ,, $1''=l_2$,,
AE ,, u_1 ,, ,, $1''=l_3$,,
and AD ,, v_1 ,, ,, $1''=l_4$,,

then
$$\frac{y_1}{l_2}\times\frac{l_4}{v_1}=\frac{x_1}{l_1}\times\frac{l_3}{u_1}$$
$$\frac{u_1}{v_1}=\frac{x_1}{y_1}\times\frac{l_3}{l_1}\times\frac{l_2}{l_4}$$

and thus
$$\frac{x_1}{y_1}=\frac{u_1}{v_1}$$

provided that the scale-values are so chosen that

$$\frac{l_1}{l_2}=\frac{l_3}{l_4}$$

In Fig. 20 AC is parallel to FB, and AD is parallel to EB

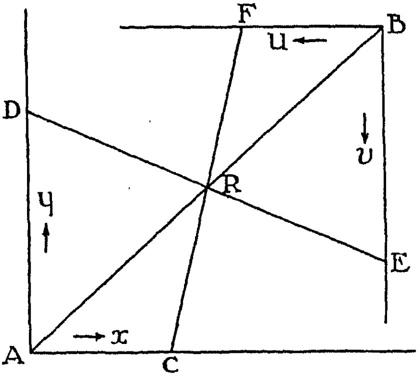


Fig. 20.

and AB is the line joining the zero markings of the four scales. Then,

if AC represents x_1 to a scale of l_1 units per inch
AD ,, y_1 ,, ,, l_2 ,, ,,
BF ,, u_1 ,, ,, l_3 ,, ,,
and BE ,, v_1 ,, ,, l_4 ,, ,,

$$AC=\frac{x_1}{l_1}\qquad AD=\frac{y_1}{l_2}\qquad BF=\frac{u_1}{l_3}\qquad BE=\frac{v_1}{l_4}$$

Now the figures ACRD and BFRE are evidently similar, so that

$$\frac{AC}{AD}=\frac{BF}{BE}$$

whence $$\frac{x_1}{l_1}\times\frac{l_2}{y_1}=\frac{u_1}{l_3}\times\frac{l_4}{v_1}$$

or $$\frac{u_1}{v_1}=\frac{x_1}{y_1}\text{ if }\frac{l_1}{l_2}=\frac{l_3}{l_4}$$

Example 5.—Construct proportional charts of the form shown in (i), Fig. 19, and (ii), Fig. 20, to illustrate the equation

$$\frac{x}{y}=\frac{u}{v}$$

when x ranges from 0 to 12, y from 0 to 5, and u from 0 to 60.

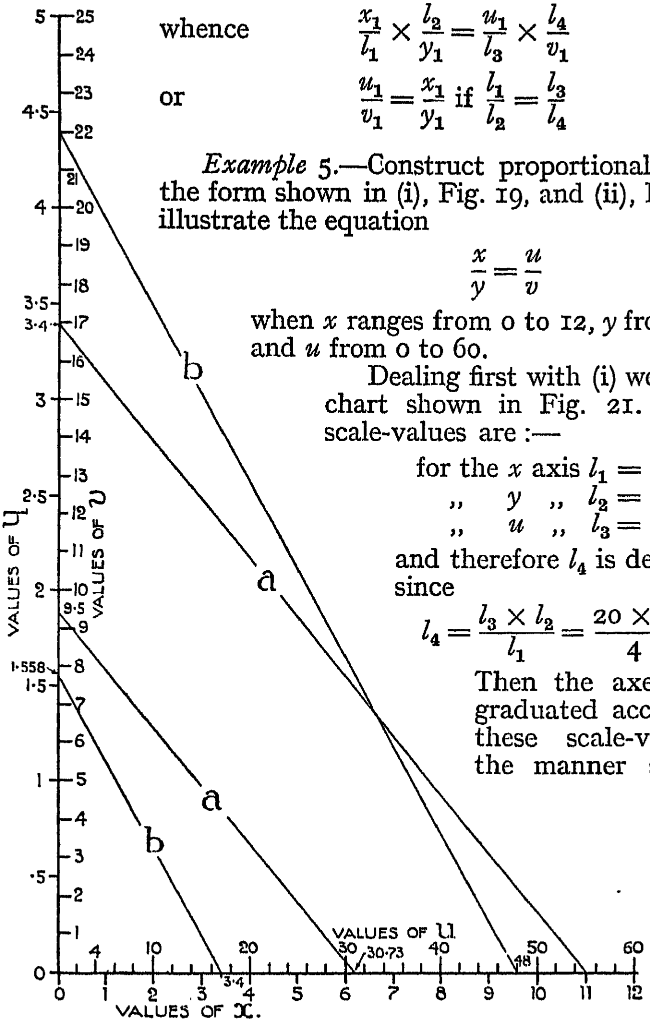
Dealing first with (i) we have the chart shown in Fig. 21. Suitable scale-values are :—

for the x axis $l_1 = 4$
,, y ,, $l_2 = 1$
,, u ,, $l_3 = 20$

and therefore l_4 is determined, since

$$l_4=\frac{l_3\times l_2}{l_1}=\frac{20\times 1}{4}=5.$$

Then the axes can be graduated according to these scale-values, in the manner shown in

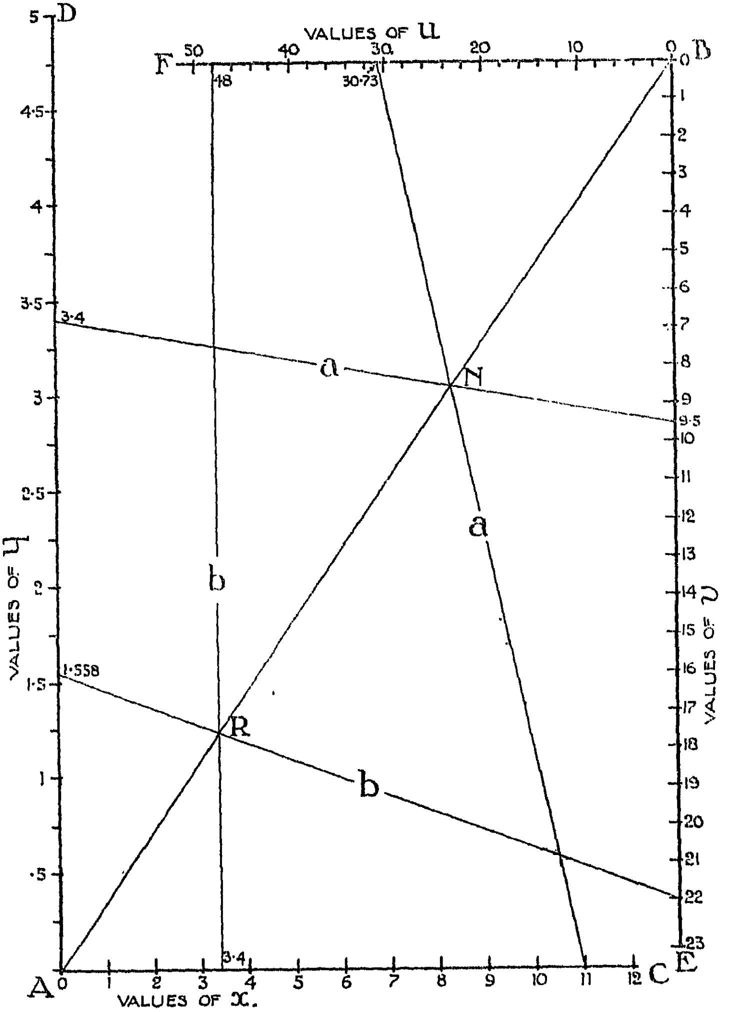


$$\frac{x}{y}=\frac{u}{v}$$

FIG. 21.

5115 4115
N 23

Fig. 21, care being taken to keep the graduations for the x scale on one side of the horizontal and those for the scale of



$$\frac{x}{y} = \frac{u}{v}$$

Fig. 22.

u on the other side, a similar precaution being taken in graduating the vertical axis.

To Use the Chart.—(*a*) Given $y = 3{\cdot}4$, $v = 9{\cdot}5$ and $x = 11$, to find u. Join 3·4 on the scale of y to 11 on the scale of x, and through 9·5 on the axis of v draw a parallel to this line to cut the axis of u in the point marked 30·73: then 30·73 is the required value of u.

(*b*) Given $u = 48$, $x = 3{\cdot}4$, $v = 22$, to find y. Join 48 on the axis of u to 22 on the axis of v, and through 3·4 on the axis of x draw a parallel to cut the axis of y at the required value 1·558.

(ii) The alternative form of chart is shown in Fig. 22. AC and AD are two axes at right angles and BF and BE are also at right angles; the distances between the pairs of parallel axes being purely arbitrary. The line AB joins the zero values of the four scales and serves as the connecting link between the two pairs of variables.

Taking the same scale-values as for case (i), the axes of x, y, u, and v are set out along AC, AD, BF, and BE respectively.

Then (*a*), to find u when $y = 3{\cdot}4$, $x = 11$ and $v = 9{\cdot}5$, join 3·4 on the axis of y to 9·5 on the axis of v, and through N, the point in which this line cuts AB, draw a line from 11 on the axis of x to cut the axis of u at 30·73.

(*b*) To find y when $u = 48$, $x = 3{\cdot}4$ and $v = 22$, join 3·4 on the axis of x to 48 on the axis of u, the line intersecting AB at R: join 22 on the v scale to R and produce the line to cut the axis of y at 1·558. It will be seen that it is essential that points on opposite axes are joined in all cases and not points on adjacent axes.

This form of chart is to be preferred to that shown in Fig. 21, since it not only tends to produce a more open figure, but it obviates the division of one line to show two scales with the consequent possibility of confusion.

Useful charts of this character can be drawn to assist in the evaluation of such a commonly recurring formula as

$$\frac{f}{y} = \frac{M}{I} = \frac{E}{R}$$

or in the application of the "sine" rule in the solution of triangles.

Chart for Solution of Triangles

Example 6.—Construct a chart suitable for the solution of triangles by the aid of the "sine" rule, viz.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

the sides having values up to 5.

Evidently it is only necessary to deal with the equation

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

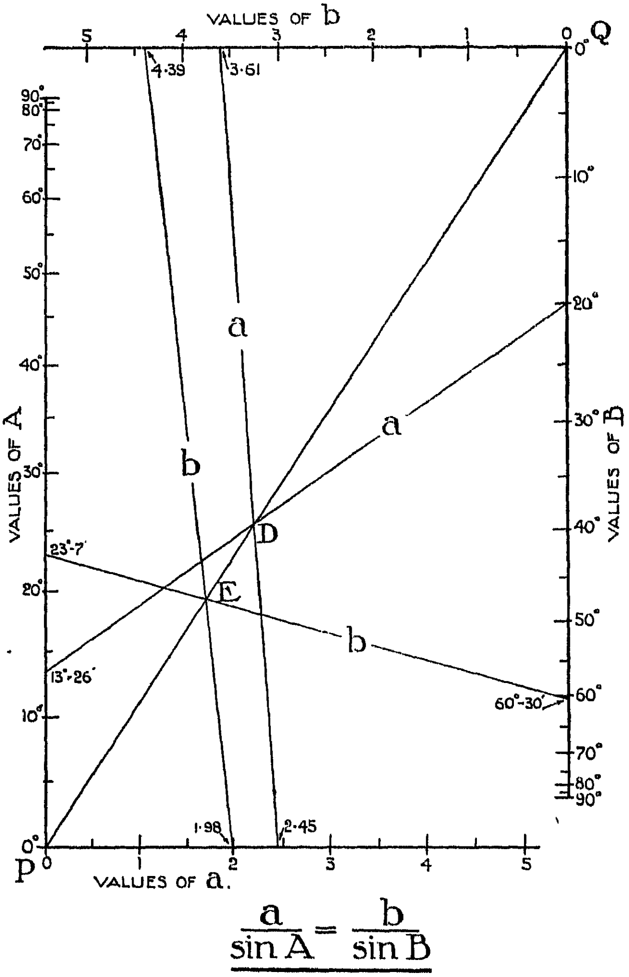


FIG. 23.—Solution of Triangles.

for in any actual example c and C can readily be written in place of b and B or a and A, as the case may demand.

The range for the angles need only be taken as 0 to

90°, because if the angle is obtuse, use may be made of the relation

$$\sin A = \sin (180 - A).$$

The maximum value of the sine is 1, since $\sin 90° = 1$, and on the diagram (Fig. 23) 10 cm. is the length representing this amount. For the lengths of sides the scale taken is $1'' = 2$ units. When setting out the scales of angles it is necessary to bear in mind that it is the magnitude of the sine that determines the length to be measured along the axis, whilst the actual marking is the value of the angle. Thus if $A = 25°$ $\sin A = \cdot4226$ and the marking for 25° is placed at a distance of $\cdot4226 \times 10$, *i. e.* 4·226 cm. from the marking for 0°.

To Use the Chart.—(*a*) Given $a = 2{\cdot}45$ inches, $b = 3{\cdot}61$ inches, and $B = 20°$; to find A. Join 2·45 on the scale of *a* to 3·61 on the scale of *b*, the line cutting PQ at D. Join 20 on the scale of B to the point D and produce the line to cut the axis of A in the point for which the marking is 13° 26′. Then $A = 13° 26'$.

(*b*) Given $c = 43{\cdot}9$ inches, $C = 119\frac{1}{2}°$, and $a = 19{\cdot}8$ inches; to find A. Join 4·39 on the scale of *b* (the *b* scale in this case serving for the *c* scale) to 1·98 on the scale of *a*, the line intersecting PQ at E. Through $60\frac{1}{2}$ on the B scale, $60\frac{1}{2}$ being the supplement of $119\frac{1}{2}°$, draw a line through E to cut the scale of A at 23° 7′: then $A = 23° 7'$.

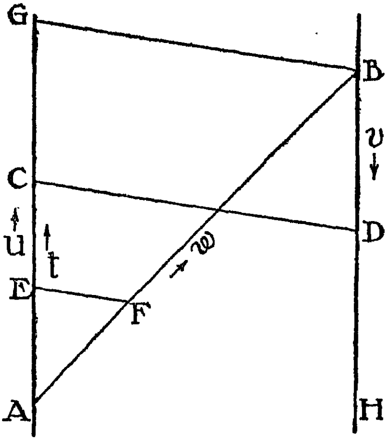


Fig. 24.

A step forward leads to a combined proportional chart as shown in Fig. 24. By means of such a chart four variables may be correlated, three axes being necessary.

In Fig. 24 let AG be the axis of u and also of another variable t, and let BH be the axis of v. Choose AB so that its length is any convenient round number of inches and let AB be utilised as the axis of a fourth variable w. Assuming that the scale-values taken in the plotting of the values of u, v, w, and t are l_1, l_2, l_3, and l_4 respectively, let AC represent some particular value of u, say u_1, AE represent t_1, AF represent w_1, and let BD represent v_1; EF being drawn parallel to CD. Then it is possible to determine the relations existing between the variables when the scale-values are connected in a particular way.

Through B draw BG parallel to CD and EF: then

$$\mathrm{AC} = \frac{u_1}{l_1} \qquad \mathrm{BD} = \frac{v_1}{l_2} \qquad \text{and } \mathrm{AB} = m.$$
$$\mathrm{AF} = \frac{w_1}{l_3} \qquad \mathrm{AE} = \frac{t_1}{l_4}$$

From the similarity of the triangles AGB and AEF

$$\frac{\mathrm{AG}}{\mathrm{AB}} = \frac{\mathrm{AE}}{\mathrm{AF}}$$

$$i.\,e.\quad \frac{\mathrm{AC} + \mathrm{BD}}{m} = \frac{\mathrm{AE}}{\mathrm{AF}}$$

$$\frac{\frac{u_1}{l_1} + \frac{v_1}{l_2}}{m} = \frac{t_1}{l_4} \times \frac{l_3}{w_1}$$

$$\frac{l_2 u_1 + l_1 v_1}{m l_1 l_2} = \frac{t_1 l_3}{l_4 w_1}$$

Now if $l_1 = l_2$, the equation becomes

$$\frac{u_1 + v_1}{m l_1} = \frac{t_1 l_3}{l_4 w_1}$$

and if the scale-values be so chosen that $m l_1 = l_4 \div l_3$

$$i.\,e.\quad m = \frac{l_4}{l_1 l_3}$$

then
$$u_1 + v_1 = \frac{t_1}{w_1}$$

or, in general,
$$u + v = \frac{t}{w}$$

where u, v, t, and w are functions.

In practice, it is usual to decide upon a convenient value for m and to select l_1 and l_3, and then l_4 is calculated from the relation $l_4 = m l_1 l_3$. The equation as it stands does not appear to be of a common type, but it must be remembered that the letters may represent functions: thus, for example, the equation might take the form $a^3 - b^3 = \frac{\sqrt{c}}{d^2}$, in which case the scales along the various axes would not be equally divided.

Chart giving Volume for Hollow Cylinder

Example 7.—Construct a chart to represent the equation for the volume of a hollow cylinder

$$V = \pi h(R^2 - r^2)$$

R ranging from 0 to 5 inches, r from 0 to 4 inches, and h from 0 to 60 inches.

The equation $\quad V = \pi h(R^2 - r^2)$

may be re-written as $\quad R^2 - r^2 = \dfrac{V}{\pi h}$

which is of the form $\quad u + v = \dfrac{t}{w}$

the difference between the two forms in the sign on the left-hand side of the equation being observed in the marking of the scale of r in an upward, instead of in a downward, direction.

To Choose the Scale-values.—

R varies from 0 to 5
R^2 „ „ 0 to 25

and if 5″ represents 25 units, then $l_1 = 5$, this being the scale along both the R and r axes.

Now h varies from 0 to 60 so that πh ranges from 0 to 188·4: a convenient scale is $1'' = 20\pi$; *i.e.* $1'' = 20\pi$ units for πh and hence $1'' = 20$ for h and $l_3 = 20\pi$.

Again, $\quad l_4 = ml_1l_3 = m \times 5 \times 20\pi$

so that if m is taken as $\dfrac{10}{\pi}$, *i. e.* 3·18″

then $\quad l_4 = \dfrac{10}{\pi} \times 5 \times 20\pi = 1000.$

To Construct the Chart.—Draw two vertical lines, AC and BD (Fig. 25), 3 inches apart and make AB of length 3·18 inches. Along AC set out a scale of R^2 and along BD a scale of r^2, taking $l_1 = 5$ in each case and numbering the axes as previously described. Along AC also mark a scale for values of V, the scale being 1000 units per inch. From A to B, along AB mark the scale of values of h, taking $1'' = 20$ units.

To Use the Chart.—(*a*) Given $R = 4''$, $r = 2''$, and $h = 40$, to find V. Join 4 on the axis of R to 2 on the axis of r and through 40 on the axis of h draw a parallel to this line to cut the axis of V at 1508: then V = 1508 is the required volume.

(*b*) If the volume = 2750 cu. in., the length = 57″, and the external radius is 5″, find the internal radius. Join 2750 on the axis of V to 57 on the axis of h, and draw a parallel to this line through 5 on the axis of R to cut the r axis at 3·106 : then the internal radius is 3·106″.

(*c*) If R = 4·8″, r = 1·3″ and the volume is 3000 cu. in.,

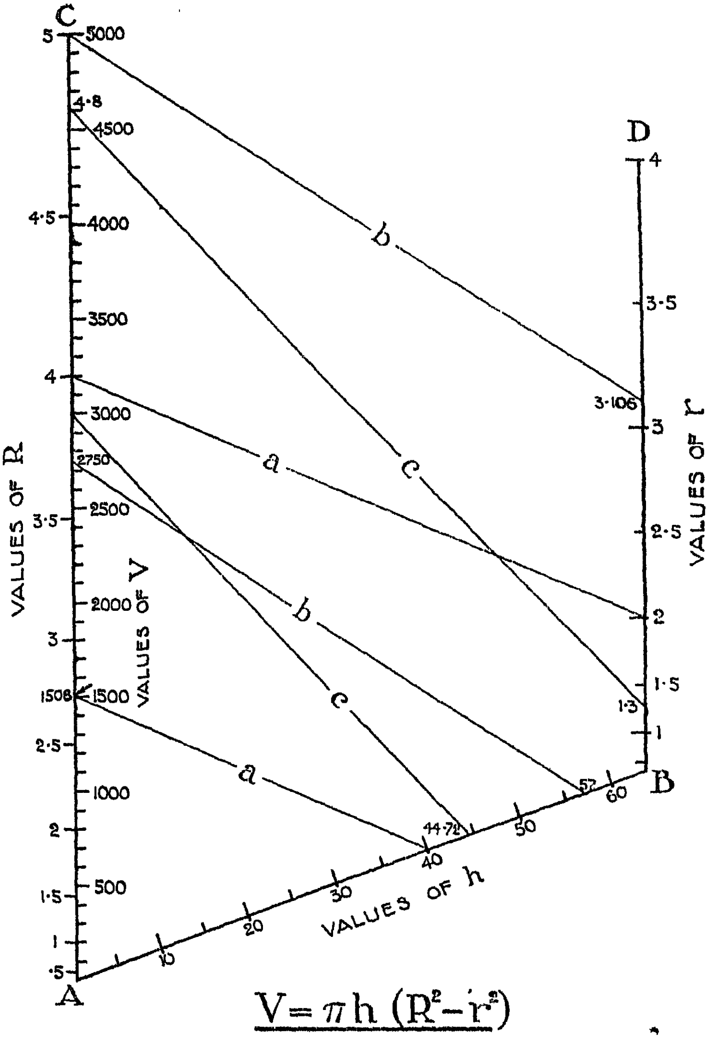


FIG. 25.—Volume of a Hollow Cylinder.

to find the length. Join 4·8 on AC to 1·3 on BD and through 3000 on the scale of V draw a parallel to cut AB in the required value, viz. 44·72.

CHAPTER III

CHARTS WITH NATURAL SCALES, BUT WITH ONE AXIS CURVED.

CHARTS in which the axes are not all rectilinear may be constructed and often prove of service, particularly in the solution of equations. The chief disadvantage of this form of chart is the large amount of calculation necessary for the graduation of the curved axis; this type of chart may, however, be adapted to assist in the solution of many forms of equations.

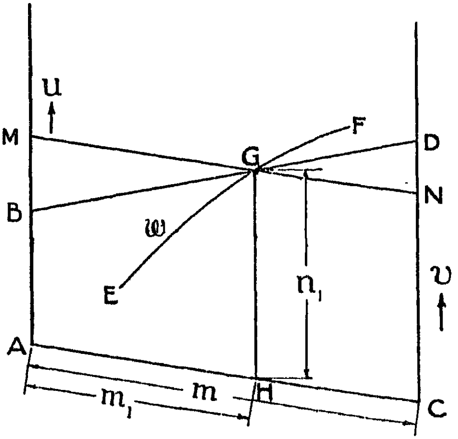


FIG. 26.

In Fig. 26, let AB and CD be the axes of u and v respectively, and let the curve EGF be the axis of w, a function connected with the other variables in a particular way. Then let it be granted that $u + vw = w'$ it being clearly understood that u, v, w, and w' all represent functions, w' and w each denoting a different function of the same variable. For example, the type of equation might be $3x + yz = 7z^4$, or the more difficult form $5u^3 + 6v^2w = w^{1.7}$.

Let AC be any convenient length, m inches, G be any point on the curve, and let MGN be drawn parallel to AC, and GH parallel to AB and CD. Also let AB represent u_1, some particular value of u, to a scale of $1'' = l_1$ units, and let CD represent the corresponding value v_1 to a scale of $1'' = l_2$ units.

Then $\text{AB} = \dfrac{u_1}{l_1}$ and $\text{CD} = \dfrac{v_1}{l_2}$

From the similarity of the triangles BGM and NGD

$$\frac{BM}{MG} = \frac{ND}{NG}$$

i. e.
$$\frac{GH - AB}{MG} = \frac{CD - GH}{MN - MG}$$

and if m_1 is written for MG and n_1 for GH,

$$\frac{n_1 - AB}{m_1} = \frac{CD - n_1}{m - m_1}$$

Substituting $\frac{u_1}{l_1}$ for AB and $\frac{v_1}{l_2}$ for CD,

$$\frac{n_1 - \frac{u_1}{l_1}}{m_1} = \frac{\frac{v_1}{l_2} - n_1}{m - m_1}$$

Thus
$$\frac{l_1 n_1 - u_1}{l_1 m_1} = \frac{v_1 - l_2 n_1}{l_2(m - m_1)}$$

$$l_1 l_2 n_1 (m - m_1) - u_1 l_2 (m - m_1) = l_1 m_1 v_1 - l_1 l_2 m_1 n_1$$

$$- u_1 l_2 (m - m_1) - v_1 l_1 m_1 = - l_1 l_2 m_1 n_1 - l_1 l_2 n_1 m + l_1 l_2 m_1 n_1$$

or
$$u_1 l_2 (m - m_1) + v_1 l_1 m_1 = l_1 l_2 m n_1.$$

Dividing through by $l_2(m - m_1)$,

$$u_1 + v_1 \frac{l_1 m_1}{l_2(m - m_1)} = \frac{l_1 n_1 m}{(m - m_1)}$$

This equation is approaching the form $u + vw = w'$ and can be made identical with it if w_1 be written in place of $\frac{l_1 m_1}{l_2(m - m_1)}$ and w_1' in place of $\frac{l_1 n_1 m}{(m - m_1)}$ (w_1 and w_1' being the particular values of w and w' directly associated with the particular values u_1 and v_1 of u and v).

Thus
$$\frac{l_1 m_1}{l_2(m - m_1)} = w_1$$

$$l_1 m_1 = w_1 l_2 m - w_1 l_2 m_1$$

$$m_1 (l_1 + w_1 l_2) = w_1 l_2 m$$

or
$$m_1 = \frac{m l_2 w_1}{l_1 + l_2 w_1} = \frac{m w_1}{\frac{l_1}{l_2} + w_1}$$

so that for any particular value of w the actual distance m_1 can be calculated, since m, $\frac{l_1}{l_2}$, and w_1 would all be known.

Similarly, an expression can be found for n_1, for

$$\frac{l_1 n_1 m}{m - m_1} = w_1'$$

whence
$$n_1 = \frac{w_1'(m - m_1)}{l_1 m}$$

but
$$m_1 = \frac{m w_1}{\frac{l_1}{l_2} + w_1}$$

so that
$$(m - m_1) = m - \frac{m w_1}{\frac{l_1}{l_2} + w_1}$$

$$= \frac{m\frac{l_1}{l_2} + m w_1 - m w_1}{\frac{l_1}{l_2} + w_1}$$

$$= \frac{m\frac{l_1}{l_2}}{\frac{l_1}{l_2} + w_1}$$

Hence
$$n_1 = \frac{w_1' m \frac{l_1}{l_2}}{l_1 m\left(\frac{l_1}{l_2} + w_1\right)} = \frac{w_1'}{l_1 + l_2 w_1}$$

Thus, knowing m_1 and n_1 for each particular value of w and w', we can obtain a series of points like G, and the curve through them is the axis on which must be marked the values of w.

The equation $u + vw = w'$ occurs frequently, for u, v, w, and w' represent functions and not the first powers of letters. As one particular illustration we can adapt this form of chart to solve the quadratic equation.

Chart for the Solution of Quadratic Equations

It has been previously shown * that the general quadratic equation can be written as $Ax^2 + Bx + C = 0$, the roots of the equation being given by the formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

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* *Mathematics for Engineers*, Part I, p. 64.

Now if α and β be written for the roots, *i. e.* if

$$\alpha = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad \beta = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

then $$\alpha + \beta = \frac{-B + \sqrt{B^2 - 4AC} - B - \sqrt{B^2 - 4AC}}{2A} = \frac{-2B}{2A}$$
$$= -\frac{B}{A}$$

and $$\alpha\beta = \frac{(-B + \sqrt{B^2 - 4AC})(-B - \sqrt{B^2 - 4AC})}{4A^2}$$
$$= \frac{B^2 - B^2 + 4AC}{4A^2} = \frac{C}{A}$$

The equation $Ax^2 + Bx + C = 0$ may be divided through by A giving
$$x^2 + \frac{B}{A}x + \frac{C}{A} = 0$$
whence, by substitution from the above,
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$
or
$$x^2 - Sx + P = 0$$
where S stands for the sum of the roots, and P for their product: and it is with the equation in this form that we shall work.

If $x^2 - Sx + P = 0$ then $x^2 = Sx - P$, this evidently being of the type $w' = vw + u$, x^2 taking the place of w', x taking the place of w, S that of v, and $-$ P that of u. Then the actual procedure for the solution of the equation can best be followed through a numerical example.

Example 1.—Construct a chart for the equation
$$x^2 - Sx + P = 0,$$
the values of S and P each ranging from -20 to $+20$.

Draw two vertical lines (Fig. 27) 3 inches apart, and from some convenient horizontal line as the line of the zero values set out scales for P and S along these axes, in opposite directions because $-$ P takes the place of u, using the scale-value 1″ = 10 units.

Thus $$m = 3,\ l_1 = l_2 = 10, \text{ and } \frac{l_1}{l_2} = 1.$$

Proceed to calculate the co-ordinates of the points on the curve which is to be the axis of x, in the following manner :—

$$m_1 = \frac{mx}{\frac{l_1}{l_2} + x} \quad \text{and} \quad n_1 = \frac{x^2}{l_1 + l_2 x}$$

i. e. $$m_1 = \frac{3x}{1 + x} \quad \text{and} \quad n_1 = \frac{x^2}{10 + 10x}$$

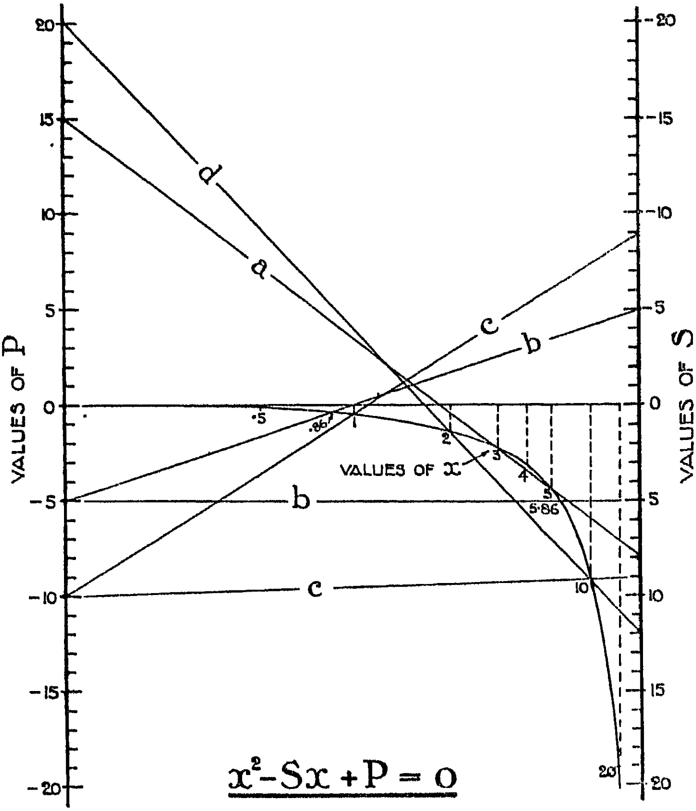

FIG. 27.—Solution of Quadratic Equations.

The table of values is calculated thus :—

x	$1 + x$	m_1	x^2	$10 + 10x$	n_1
·5	1·5	1	·25	15	·017
1	2	1·5	1	20	·05
2	3	2	4	30	·133
3	4	2·25	9	40	·225
4	5	2·4	16	50	·32
5	6	2·5	25	60	·417
10	11	2·73	100	110	·91
20	21	2·86	400	210	1·91
30	31	2·91	900	310	2·91
40	41	2·93	1600	410	3·92
50	51	2·95	2500	510	4·92

Plot these values, those of m_1 along the horizontal line and towards the right, and those of n_1 vertically downward, marking each point so obtained with the value of x for which it was determined; and draw in a smooth curve through this series of points, this being the axis of x.

To Use the Chart.—(*a*) Let it be required to solve the equation

$$x^2 - 8x + 15 = 0.$$

Comparing this equation with the standard form, we see that $S = 8$ and $P = 15$. Hence join $S = 8$ to $P = 15$ and note the points at which this line cuts the axis of x, viz. at $x = 3$ and $x = 5$: then the roots of the equation are 3 and 5.

(*b*) To solve the equation $x^2 + 5x - 5 = 0$. In this case $S = -5$, and $P = -5$, and two lines are drawn on the diagram, although one would suffice. Joining -5 on the axis of S to -5 on the axis of P, we have a line which intersects the curve at the point where $x = \cdot 86$. By joining $+5$ on the scale of S to -5 on the scale of P, we find the intersection with the curve at $5\cdot 86$; but since to obtain this line we changed the sign before S, we must change the sign before this value, calling the roots $\cdot 86$ and $-5\cdot 86$.

Actually the second line is unnecessary if use is made of the fact that S is the sum of the roots, so that if one root is found to be $\cdot 86$, and the sum of the roots is known to be -5, then the other root must be $-5\cdot 86$.

(*c*) To solve the equation $x^2 - 9x - 10 = 0$. Here $S = 9$ and $P = -10$ and the line (*c*) joining these particular values gives, by its intersection with the curve, the positive root 10. By altering the value of S to -9 and joining this point to $P = -10$ we find a value of x, viz. $+1$, from which we can say, according to the reasoning given above, that the roots are $+10$ and -1.

(*d*) To solve the equation $x^2 + 12x + 20 = 0$. In this case the roots are obviously both negative, since their sum is -12 and their product is $+20$. Thus we must proceed to find the positive values of x (in reality the roots of the equation $x^2 - 12x + 20 = 0$, which are equal *numerically* to those of the equation $x^2 + 12x + 20 = 0$) and then change both the signs. Thus although $S = -12$ and $P = 20$, we join $S = +12$ to $P = 20$ to cut the curve at $x = 10$ and 2, and write down the solutions as $x = -10$ or -2.

Thus the various forms of equation have been considered and it may be advisable to repeat the procedure to be followed under the various conditions, in the form of a summary :—

Case 1.—S and P both positive. Join S to P and note the two points of intersection with the axis of x. If the roots are equal the line touches the curve at one point only: and if S is less than $\sqrt{4P}$ the line does not cut the curve at all, the roots being imaginary.

Case 2.—S and P both negative. Join the S and P values as given and note the point of intersection with the axis of x: this is the value of one root and subtraction from S gives the other root.

Case 3.—S positive and P negative. Join S to P and note the value at the intersection with the axis of x: this is the positive root and by subtracting it from S the other root is found. As an alternative method, join $-$ S to P and so obtain the negative root.

Case 4.—S negative and P positive. Join $-$ S to P, note the points of intersection and place the minus sign before each of these values.

A nomogram for the solution of quadratic equations, but of a rather different type, may be interpolated here; it is, however, by no means so useful as the type just discussed.

It has been shown * that the equation $Ax^2 + Bx + C = 0$ can be solved by plotting the curve $y = Ax^2 + Bx + C$ and noting the values of x at the points of intersection of the curve and the x axis. It is equally good to write the equation as $Ax^2 = -Bx - C$, to plot the parabola $y_1 = Ax^2$ and the straight line $y_2 = -Bx - C$, and to note the x values at the points of intersection of curve and line. By a slight modification in the placing of the graduations this latter method can be transformed into a nomogram for the solution of quadratic equations.

As shown on p. 40 the general quadratic may be written as $x^2 - Sx + P = 0$

whence $$x^2 = Sx - P$$

which equation can be solved by determining the intersections of the parabola $y_1 = x^2$ and the straight line $y_2 = Sx - P$, the slope of which is S and the intercept on the vertical axis through the origin is $-$ P.

To adapt this diagram to the form of nomogram desired, it is first necessary to mark the points on the curve $y_1 = x^2$ with the x values, instead of marking the x values along the horizontal axis. Thus, for example, the point (2, 4) would simply be denoted as the point at which the marking 2 is placed, and the graduation of the curve would be made by erecting verticals from the x axis to cut the curve (Fig. 29).

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* *Mathematics for Engineers*, Part I, p. 176.

Along the vertical axis is indicated the scale used for y_1 in the construction of the parabola, but with the minus signs placed above the horizontal axis instead of below it, this vertical axis so graduated serving as the axis of P. Through the 1 on the original horizontal scale, and thus through 1 on the parabola, is drawn a vertical axis, along which are marked the values of S, the same scale-value being used as for the axis of P, but the positive values being above the horizontal axis.

Then it is evident that a line joining the origin to some marking on the S scale has a slope of S, since it rises or falls an amount S for a unit change in a horizontal direction. Also, if a line parallel to this line is drawn through a marking P, this line must have a slope of S and an intercept of $-$ P, since the graduations along the P axis are positive downwards instead of upwards. Hence the intersection or intersections of this line with the curve give the solution or solutions of the equation $x^2 - Sx + P = 0$. Thus in Fig. 28 ADE is the curve representing the equation $y = x^2$, AF = 1 horizontal unit, and CDE is parallel to AB. Now if FB represents some value of S and AC some value of P, the slope of $AB = \frac{BF}{AF}$ $= \frac{S}{1} = S$ and the intercept of the parallel line CDE is, according to the ordinary convention, $-$ P, so that the equation to the line CDE is $y = Sx - P$, and the x values at D and E are the roots of the equation $x^2 - Sx + P = 0$.

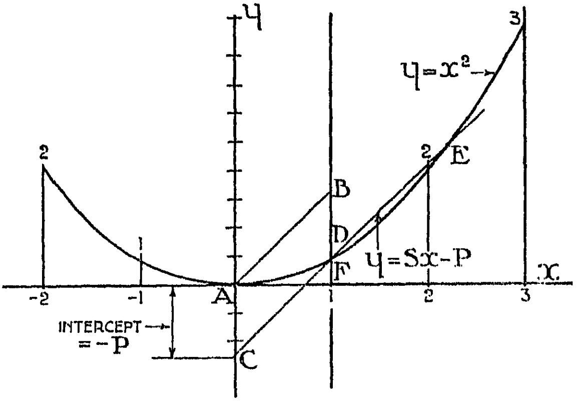


FIG. 28.

To illustrate this method more clearly, *Example* 1, p. 40, is repeated. In Fig. 29 the axes of P and S are parallel, and

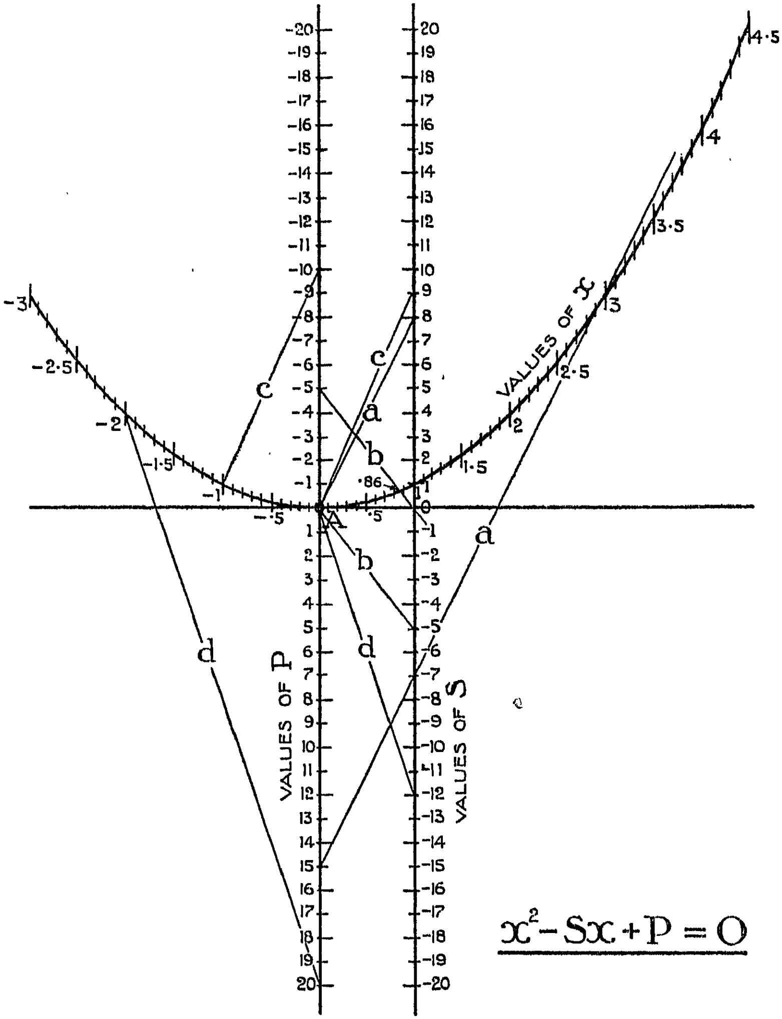


FIG. 29.—Solution of Quadratic Equations.

are graduated to the same scale, but in opposite directions, and the curve $y = x^2$ is shown plotted for values of x ranging from − 3 to + 4·5, these values of x being marked along the curve.

Then to Use the Chart.—(*a*) To solve the equation $x^2 - 8x + 15 = 0$. Here S = 8 and P = 15: hence join the point A to 8 on the scale of S and through P = 15 draw a parallel to this line to cut the curve at $x = 3$. The diagram is not extended enough to show the other point of intersection, viz. at $x = 5$, but it is not really necessary that this should be so, for if the sum of the roots is 8, and one root is 3, the other must be 5.

(*b*) To solve the equation $x^2 + 5x - 5 = 0$. In this case S = − 5 and P = − 5. By joining A to S = − 5 and then drawing a parallel through P = − 5, the value $x = \cdot 86$ is found at the intersection of line and curve. Thus the two roots are ·86 and − 5 − ·86, *i. e.* − 5·86.

(*c*) To solve the equation $x^2 - 9x - 10 = 0$. Here S = 9 and P = − 10, and the procedure is indicated by the lines (*c*) on the diagram, the intersection with the curve being at $x = -1$. Thus the two roots are − 1 and 9 − (− 1), *i. e.* 10.

(*d*) To solve the equation $x^2 + 12x + 20 = 0$. Here S = − 12 and P = 20 and a solution $x = -2$, is indicated by the lines (*d*), the other root being − 12 − (− 2), *i. e.* − 10.

It will be seen that these methods for the solution of equations can be used for equations other than quadratics, provided that the equation has the form $F(x) + mx + c = 0$, where $F(x)$ is any function of x.

A useful example of the type of chart discussed on p. 37 concerns the equation connecting the diameter of a circle with the length of chord and the height of arc.

Referring to Fig. 30, in which a = half the chord of the arc, h = height of arc, and d = diameter of circle,

$$\left(\frac{d}{2}\right)^2 = a^2 + \left(\frac{d}{2} - h\right)^2$$

$$\frac{d^2}{4} = a^2 + \frac{d^2}{4} + h^2 - dh$$

whence $$dh - a^2 = h^2$$

which is of the form $vw + u = w'$,

so that a chart can readily be constructed.

Chart giving Length of Chord of Circle.

Example 2.—Construct a chart to give values of the variables in the equation $a^2 + h^2 = dh$, a ranging from 0 to 10, and d from 0 to 50.

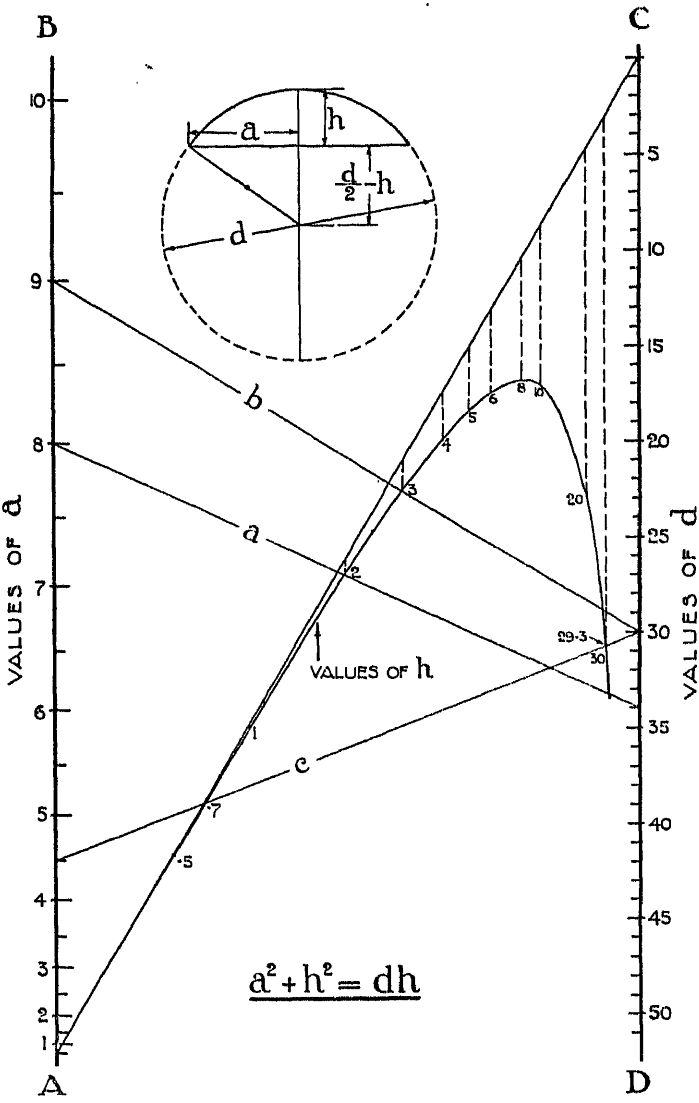


FIG. 30.—Length of Chord of a Circle.

The transposed equation is $dh - a^2 = h^2$, so that d takes the place of v, h the place of w, $-a^2$ the place of u, and h^2 replaces w'.

Draw the parallel axes AB and CD (Fig. 30) 3 inches apart. (This distance is chosen as a matter of convenience and has no significance.) Regarding A as the position of the zero of the scale of a, draw the line AC to cut the axis CD, its length being exactly 6 inches: thus $m = 6$. Now a varies from 0 to 10, so that a^2 has a maximum value of 100: hence set out along AB a scale of a^2, taking the scale-value $l_1 = 20$. From C, but in a downward direction, set out the scale of d, an evenly divided scale, with $l_2 = 10$.

Then, substituting for w_1 and w_1' their particular values h_1 and h_1^2, we have

$$m_1 = \frac{mh_1}{\frac{l_1}{l_2} + h_1}$$

and

$$n_1 = \frac{h_1^2}{l_1 + l_2 h_1}$$

and, since $l_1 = 20, \quad l_2 = 10, \quad \text{and } m = 6$

$$m_1 = \frac{6h_1}{2 + h_1}$$

$$n_1 = \frac{h_1^2}{20 + 10h_1}$$

and the tabulation of a few values is as follows :—

h_1	h_1^2	$20 + 10h_1$	m_1	n_1
·5	·25	25	1·2	·01
·7	·49	27	1·56	·018
1	1	30	2	·033
2	4	40	3	·1
3	9	50	3·6	·18
4	16	60	4	·266
5	25	70	4·28	·359
6	36	80	4·5	·45
8	64	100	4·8	·64
10	100	120	5	·833
20	400	220	5·46	1·82
30	900	320	5·64	2·813

Plot the values of m_1 from A along AC and the values of n_1 vertically downward; draw a smooth curve through the points so obtained and mark the various points for the values of h from which they were calculated.

To Use the Chart.—(a) Given the length of the chord = 16″, and the height of the arc = 2″, to find the diameter of the circle.

Here $a = 8$ and $h = 2$: hence join 8 on the axis of a to 2 on the curve and produce the line to cut the axis of d at 34. Then 34 inches is the required diameter.

(*b*) Given that $h = 3''$ and $d = 30''$, to find a. Join 30 on the axis of d to 3 on the curve and produce the line to cut the axis of a at the required value, viz. 9.

(*c*) Given that $d = 30''$ and $a = 4{\cdot}5''$, to find h. In cases where a value of h is sought, it must be remembered that the chart should give the two possible values of h, viz. the solutions of the quadratic equation $h^2 - dh + a^2 = 0$; the heights being respectively that of the arc greater, and that of the arc less, than a semicircle. Thus the line joining $d = 30$ and $a = 4{\cdot}5$ cuts the curve at the two points for which $h = {\cdot}7$ and $h = 29{\cdot}3$, the sum of these results being equal to the length of the diameter.

CHAPTER IV

CHARTS INVOLVING LOGARITHMIC PLOTTING.

THE charts illustrating the general equation $w = u + v$ and the modifications of it already indicated, for which the plotting has been of natural scales only, have a limited usefulness. In the majority of formulæ of value to the engineer, products and quotients of the variable quantities occur, whilst in many instances also the products are of powers of the variables. It is therefore essential that the method of construction of charts suitable for these conditions should be explained; and it will be shown that the fundamental change in the procedure is that natural scales are replaced by logarithmic scales.

Thus, if $ab = c$ $\quad \log a + \log b = \log c$
or $\quad u + v = w$

if u, v, and w are written in place of the logs of a, b, and c respectively, and consequently the chart takes the form of three parallel axes all carrying logarithmic scales.

Charts involving logarithmic plotting are in general more difficult to construct than those with natural scales, but, if the work on functional scales (see Chapter I) has been carefully followed, the fundamental trouble has been removed. The chief objection to these charts is the amount of labour and time expended in reading the logs from the tables and the conversion of the logs to actual distances according to the particular scale-values chosen. Much of the time, however, can be saved by the construction to a large scale of the diagram shown in Fig. 31.

A horizontal line AB is drawn of some convenient length: in Fig. 31 AB = 5 inches. The vertical axis BC is carefully and minutely graduated, the scale being logarithmic and the scale-value l being chosen from a consideration of the space available and the size of charts for which the scales are to be used. In the diagram, l is taken as ·3, *i.e.* 1″ represents ·3 unit of logs, or $3\frac{1}{3}$ inches represent 1 unit of logs, *i.e.* the distance from the marking for 1 to the marking for 10. The length AB is divided into a number of equal parts and verticals are erected through the points of division. Then the diagonal line AC gives by its intersections with these verticals the

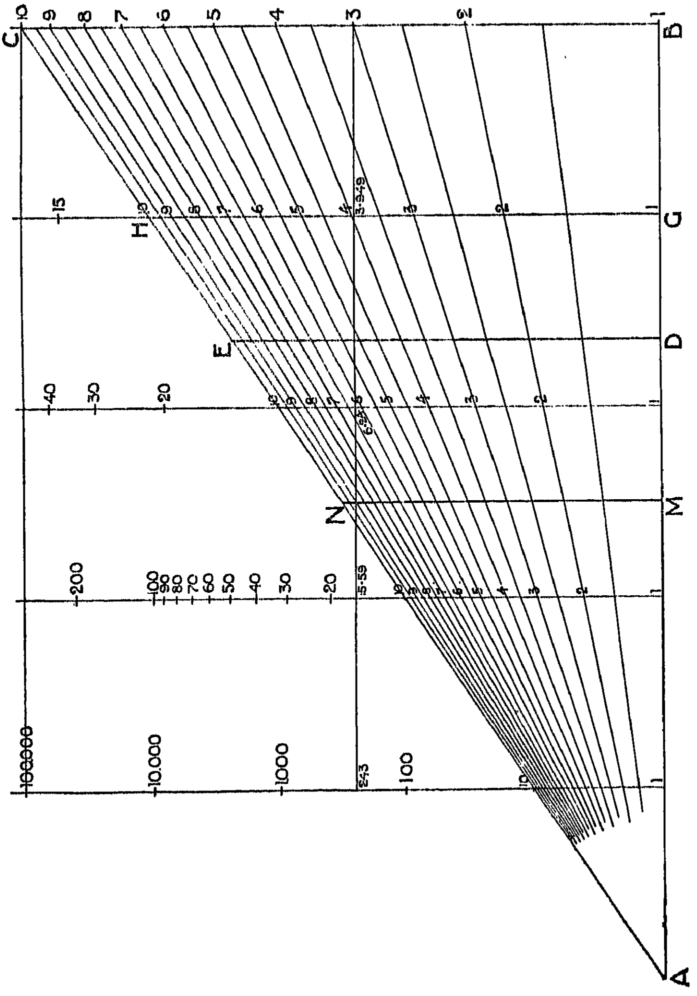


FIG. 31.

positions of the markings for the 10 of each axis, and in like manner any other subdivisions can be obtained.

If DE be any vertical $\frac{DE}{AD} = \frac{BC}{AB}$

also if l_x is the scale-value along DE, $\mathrm{DE} = \frac{1}{l_x}$

so that
$$\frac{1}{l_x \times \mathrm{AD}} = \frac{1}{l \times \mathrm{AB}}$$

whence
$$l_x = l \times \frac{\mathrm{AB}}{\mathrm{AD}}$$

Thus, for the vertical GH, where AG = 4 inches

$$l_x = \frac{\cdot 3 \times 5}{4} = \cdot 375$$

whilst if a scale-value of ·6 unit per inch is desired, it is necessary to draw the vertical MN so placed that

$$\mathrm{AM} = \frac{\mathrm{AB} \times \cdot 3}{\cdot 6} = \cdot 5\mathrm{AB} = 2\tfrac{1}{2} \text{ inches.}$$

Fig. 31 can also be utilised to give powers and roots: thus if a horizontal line be drawn through 3 on the axis BC, it cuts the consecutive equidistant verticals at the points 3·949, 6·24, 15·59, and 243, which are respectively the values of $3^{\frac{5}{4}}$, $3^{\frac{5}{3}}$, $3^{\frac{5}{2}}$, and 3^5. Incidentally, this horizontal shows that

$$15 \cdot 59 = 3^{\frac{5}{2}} = \sqrt{3^5} = \sqrt{243}.$$

It is probably more convenient in practice to space the verticals so that the scale-values are suitable, rather than to erect the ordinates at equal intervals. The base line is then graduated according to the scale-values, as shown in Fig. 32, the method of construction of which should be apparent from the reasoning already given.

Occasionally the semi-logarithmic paper which can be bought may be used with advantage in the construction of these logarithmic charts. This paper is divided in one direction according to a logarithmic scale, and in the direction at right angles the divisions are equal. To derive the greatest benefit from the use of this paper it is desirable to employ it in cases in which it is convenient to use the same scale on the outside axes. Thus it would be very suitable for the chart $uv = w$ if the same scale-value was adopted for both the u and the v axes.

It is now possible to proceed to the construction of charts to assist in multiplication of functions of the variable quantities; and instead of taking an example of the most elementary kind, *i. e.* one in which the equation is of the simple form $w = uv$ (a worked example of this type can be found in *Mathematics for Engineers*, Part I, Chapter XI), we shall apply our working to a case of slightly greater difficulty. Also this first example will be worked by two methods, so that a comparison of the two forms of chart can be made.

The chart for the equation $w = uv$, or its modifications, is of exactly the same character as that for the equation $w = u + v$ in so far as concerns the relation between the scale-values

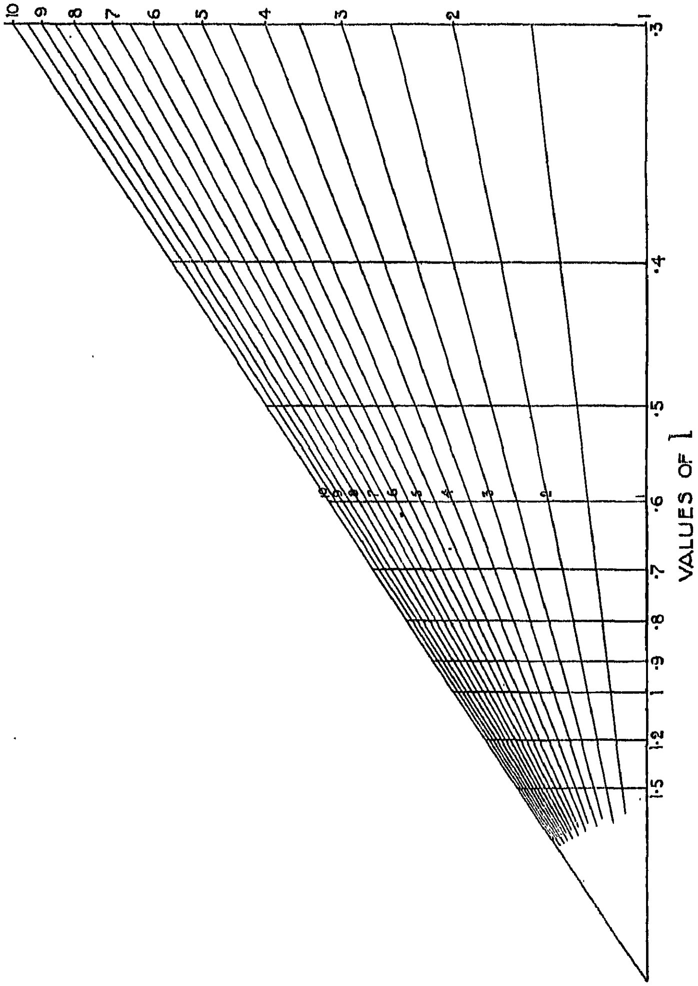


FIG. 32.

and the disposition of the axes; thus $l_3 = l_1 + l_2$ and $\frac{m_1}{m_2} = \frac{l_2}{l_1}$, but it differs by reason of the fact that the scales along the three axes are logarithmic and not natural scales.

Charts giving Weight of Silicate Grinding Wheels

Example 1.—Construct (A) an alignment chart
and (B) an intercept chart
to give the values of the variables in the formula

$$W = \frac{d^2 \times t}{13}$$

where W is the weight in lbs. (of silicate grinding wheels)
d is the diameter in inches
and t is the thickness in inches
the range of the diameter to be from 1″ to 12″ and the thickness to vary from $\frac{1}{8}$″ to $\frac{3}{4}$″.

(A) Draw the axes of d and t (Fig. 33) $3\frac{1}{2}$″ apart and graduate them according to the following scheme :—

For the axis of d, the range of values of d is from 1 to 12, so that d^2 varies from 1 to 144 and $\log d^2$, or $2 \log d$, varies from 0 to 2·1584. If 4·3168 inches be taken to represent this range the scale-value l_1 is ·5, and the table of distances is :—

d	1	2	3	4	5	6	7	8	9	10	11	12
$\log d$ . . .	0	·301	·4771	·6021	·699	·7782	·8451	·9031	·9542	1	1·0414	1·0792
$2(\log d - \log 1)$	0	·602	·9542	1·2042	1·398	1·5564	1·6902	1·8062	1·9084	2	2·0828	2·1584
Distance from A (inches) .	0	1·204	1·908	2·408	2·796	3·113	3·380	3·612	3·817	4	4·166	4·317

the figures in the fourth line being obtained from those in the third line by dividing the latter by the scale-value ·5.

For the axis of t, the range required is $\log \frac{3}{4} - \log \frac{1}{8}$, *i. e.* ·7782, and a suitable scale-value is ·2 ; hence the graduation of the axis can either be determined from a table in which the vulgar fractions are first converted to decimals, as

t	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
$\log t$ . . .	$\bar{1}$·0969	$\bar{1}$·2729	$\bar{1}$·39[illegible]	$\bar{1}$·4949	$\bar{1}$·574	$\bar{1}$·6410	$\bar{1}$·699	$\bar{1}$·7501	$\bar{1}$·7959	$\bar{1}$·8373	$\bar{1}$·8751
Difference of logs .	0	·1760	·3010	·3980	·4771	·5441	·6021	·6532	·699	·7404	·7782
Distance from B	0	·88	1·505	1·99	2·386	2·721	3·011	3·266	3·495	3·702	3·891

or use may be made of a chart of the form shown on p. 53, it being remembered that the fractions $\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{4}$, etc., are in the proportion 1, 1·5, 2, etc., so that the distance from the marking for $\frac{1}{8}$ to that for $\frac{5}{8}$, for example, is the distance from 1 to 5 on a ·2 scale.

Now $l_1 = \cdot 5$ and $l_2 = \cdot 2$, so that $l_3 = l_1 + l_2 = \cdot 7$ and $\frac{m_1}{m_2} = \frac{l_2}{l_1} = \frac{2}{5}$ or $m_1 = \frac{2}{7}$ of m, *i. e.* 1″.

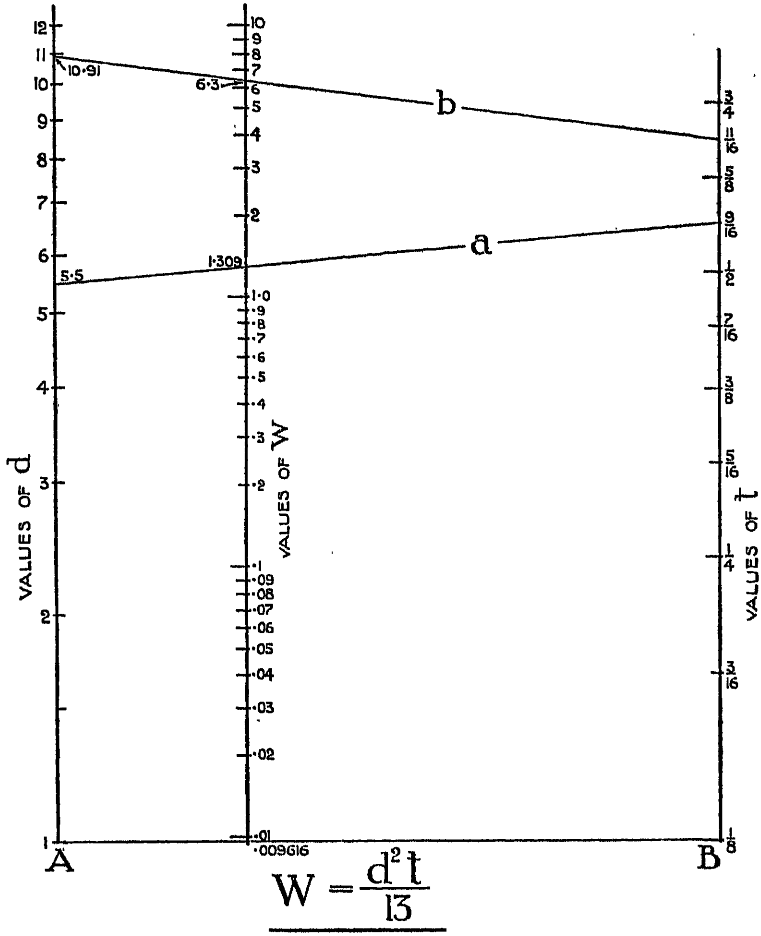


FIG. 33.—Weight of Silicate Grinding Wheels.

Join A to B and note the point in which this line intersects the axis of W; the marking to be placed here is ·009616, since $W = \frac{1^2}{13} \times \frac{1}{8}$, but it is easier to fix first the distance from this point of the marking for ·01 and then graduate the axis, taking the scale-value ·7, than to graduate directly from the ·009616 mark. The distance of the ·01 mark from the ·009616 mark is $\frac{\log \cdot 01 - \log \cdot 009616}{\cdot 7}$, *i. e.* ·0243 inch: and the graduation of the axis presents no difficulty.

To Use the Chart.—(*a*) Find the weight of a wheel of diameter $5\frac{1}{2}''$ and thickness $\frac{9}{16}''$. Join $d = 5\frac{1}{2}$ to $t = \frac{9}{16}$ to cut the axis of W at 1·309: the weight is thus 1·309 lbs.

(*b*) Find the diameter if the weight is 6·3 lbs. and the thickness is $\frac{11}{16}''$. Join $t = \frac{11}{16}$ to $W = 6{\cdot}3$ and produce to cut the axis of d at $d = 10{\cdot}91$.

(B) The form of **Intercept Chart** employed is simply a combination of two graphs, placed side by side. Its construction is simpler than that of the alignment chart, but the method of its use is more involved.

If $W = \frac{d^2 \times t}{13}$, we may write $W = M \times \frac{t}{13}$, where M is written in place of d^2.

Now plot M against d in the left-hand portion of the diagram, as shown in Fig. 34, thus obtaining a parabola. Then if t is constant W varies directly as M, so that for each value of t a straight line can be drawn to correlate W and M. The origin must be one point on each of these lines and a second point may be obtained by a simple calculation. Thus if $M = 104$, $W = \frac{104 \times t}{13} = 8t$ and consequently a series of points must be marked along the horizontal through $M = 104$: for example, if $t = \frac{11}{16}$, $8t = 5{\cdot}5$, and the line joining the origin to the point $W = 5{\cdot}5$, $W = 104$, must be marked as that for $t = \frac{11}{16}$. In the same way, other lines may be drawn.

To Use the Chart.—(*a*) To find W when $d = 5\frac{1}{2}$ and $t = \frac{9}{16}$. Erect a perpendicular through $W = 5{\cdot}5$ to meet the parabola, draw a horizontal through the point of meeting to cut the line marked $t = \frac{9}{16}$, and drop a perpendicular to the scale of W, reading off the value as 1·309.

(*b*) If W is 6·3 and $t = \frac{11}{16}$, to find d, draw a vertical through 6·3 on the scale of W to cut the line for which $t = \frac{11}{16}$, through this point of intersection draw a horizontal to intersect the curve and finally project down to the required value on the scale of d, viz. 10·91.

In this particular example the intercept chart is the easier to construct, the alignment chart is the easier to use, while there is little to choose between the two forms of chart as regards the amount of space required for the same degree of accuracy.

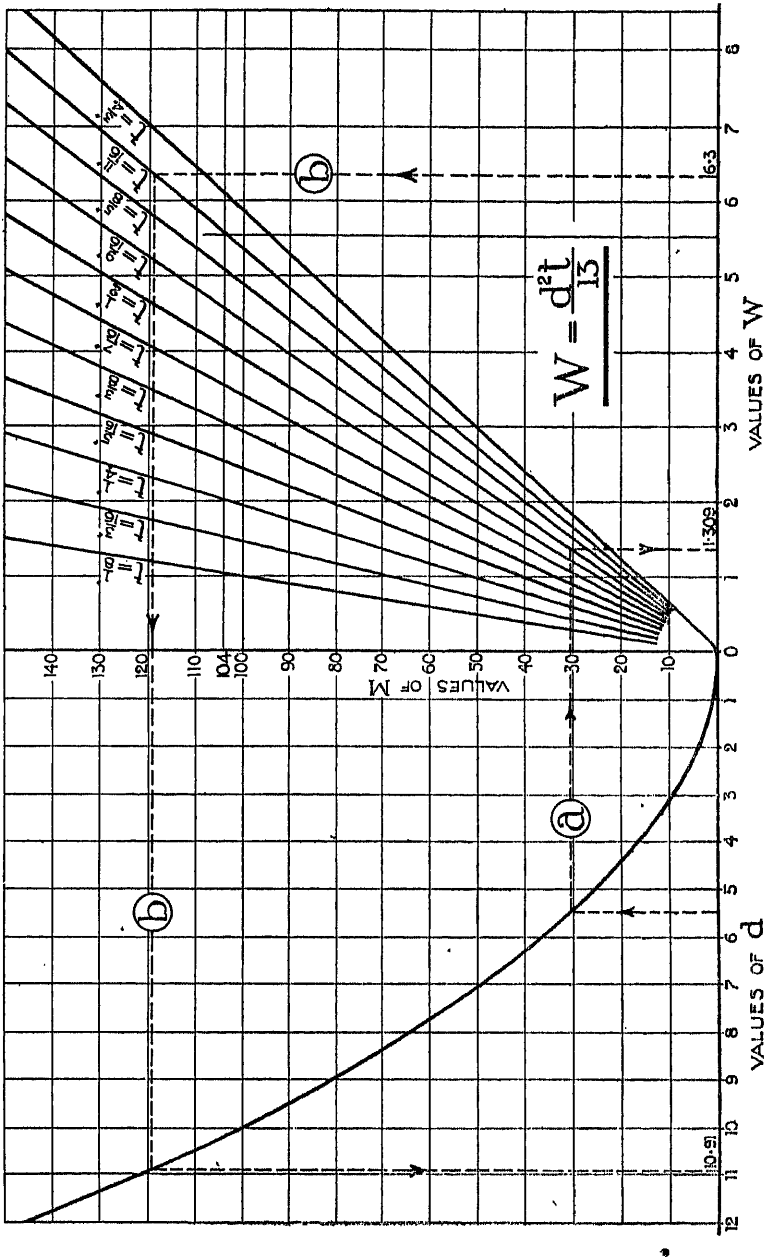


FIG. 34.—Weight of Silicate Grinding Wheels.

Chart giving Thickness of Engine Cylinder

Example 2.—If T = the thickness of the barrel of a steam engine cylinder (in inches)

D = diameter of cylinder (in inches)

and K is a constant depending upon the steam pressure, various values being

1 for a steam pressure of 160 lbs. per sq in.
·8 ,, ,, ,, 120 ,, ,,
·6 ,, ,, ,, 80 ,, ,,

then
$$T = \frac{KD}{20} + \cdot 3.$$

To design a chart to express this equation when D varies from 5 to 30 and K varies from ·2 to 1, *i. e.* the steam pressure varies from about 30 to 160 lbs. per sq. in.

It is first necessary to transform the equation so that

$$T - \cdot 3 = K \times \frac{D}{20}$$

whence, by taking logs

$$\log (T - \cdot 3) = \log K + \log \frac{D}{20}$$

Now D ranges from 5 to 30

$\frac{D}{20}$,, ,, ·25 to 1·5

and $\log \frac{D}{20}$,, ,, $\bar{1}\cdot3979$ to ·1761, *i. e.* about ·8 unit.

If we let 4″ represent ·8 unit of $\log \frac{D}{20}$ then 1″ = ·2 unit and thus $l_1 = \cdot 2$.

Again K varies from ·2 to 1 and log K from $\bar{1}\cdot301$ to 0, a range of about ·7 unit, so that the same scale-value will be convenient for this axis: thus $l_2 = \cdot 2$.

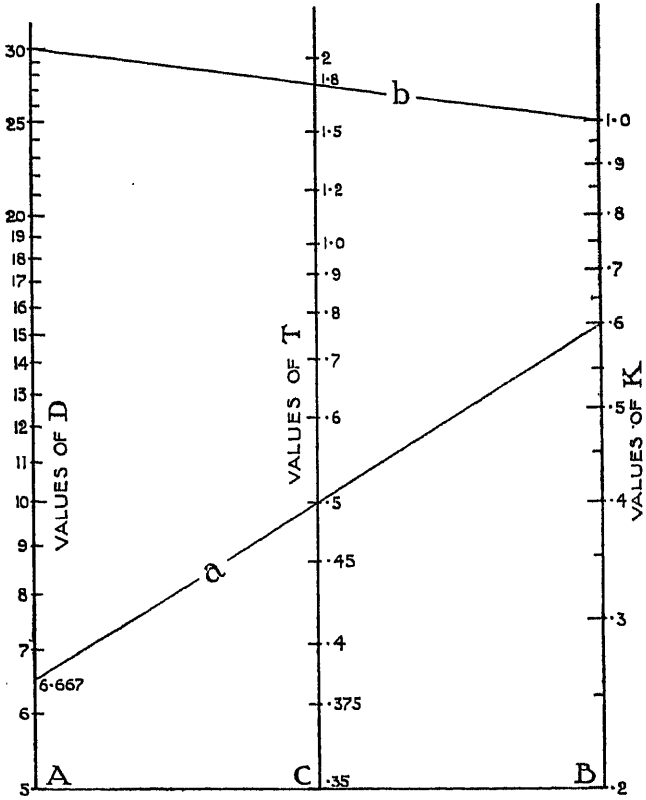
Also $l_3 = l_1 + l_2 = \cdot 4$ and $m_1 = m_2$ since $l_2 = l_1$.

The axes for D and K are drawn 3″ apart (see Fig. 35) and the axis of T is placed mid-way between them.

The manner of setting out the three scales will be understood from the tables following, in which a few points only are indicated.

For the Axis of D.—The virtual zero is at D = 5 and the distances from this marking must be calculated from the differences between the values of $\log \frac{D}{20}$ and $\log \frac{5}{20}$. It should be evident, however, that there is no real need to calculate values of $\frac{D}{20}$ and $\log\frac{D}{20}$, since

$$\log\frac{D}{20}-\log\frac{5}{20}=\log D-\log 20-\log 5+\log 20$$
$$=\log D-\log 5.$$



$$T=\frac{KD}{20}+\cdot 3$$

FIG. 35.—Thickness of Engine Cylinder.

Now $l_1=\cdot 2$ so that the actual distance from the marking at 5 to the marking at $D_1=\frac{\log D_1-\log 5}{\cdot 2}=5\times$ difference of logs, and the table of values reads :—

D	5	8	10	12	15	20	25	30
log D	·6990	·9031	1	1·0792	1·1761	1·3010	1·3979	1·4771
log D − log 5	0	·2041	·3010	·3802	·4771	·6020	·6989	·7781
Actual distance from A (inches)	0	1·0205	1·505	1·901	2·3855	3·01	3·4945	3·8905

For the axis of K, ·2 is taken as the zero marking, and l_2 = ·2, so that the distance from B of a marking K_1

$$= \frac{\log K_1 - \log \cdot 2}{\cdot 2}$$

$$= 5 (\log K_1 - \log \cdot 2)$$

and the table of values is

K	·2	·3	·4	·5	·6	·7	·8	·9	1
log K	$\bar{1}$·3010	$\bar{1}$·4771	$\bar{1}$·6021	$\bar{1}$·6990	$\bar{1}$·7782	$\bar{1}$·8451	$\bar{1}$·9031	$\bar{1}$·9542	0
log K − log ·2 . .	0	·1761	·3011	·3980	·4772	·5441	·6021	·6532	·6990
Actual distance from B (inches)	0	·8805	1·5055	1·99	2·386	2·7205	3·0105	3·266	3·495

The line AB may be regarded as the line of virtual zero values, and the value of T at its intersection with the axis of T must be found to serve as the zero for the graduation of that axis.

At A, D = 5; and at B, K = ·2,

hence
$$T \text{ (at C)} = \frac{\cdot 2 \times 5}{20} + \cdot 3 = \cdot 35,$$

so that at the point C must be placed the marking ·35 and all the distances can be calculated from C, taking the scale-value l_3 = ·4; the table of values being :—

T	·35	·5	1	1·5	1·8	2
T − ·3	·05	·2	·7	1·2	1·5	1·7
log (T − ·3) . .	$\bar{2}$·6990	$\bar{1}$·3010	$\bar{1}$·8451	·0792	·1761	·2304
log (T − ·3) − log ·05	0	·6020	1·1461	1·3802	1·4771	1·5314
Actual distance from C (inches)	0	1·505	2·865	3·451	3·693	3·829

To Use the Chart.—(*a*) If the pressure is 80 lbs. per sq. in. and the thickness is ·5″, what is the diameter of the cylinder? When the pressure is 80, the value of K is ·6; hence join ·6 on the axis of K to ·5 on the axis of T and produce the line to cut the axis of D at 6·667 : the diameter is thus 6·667 inches.

(*b*) Given the diameter = 30″ and K = 1, to find the thickness. Join 30 on the axis of D to 1 on the axis of K to intersect the axis of T at the marking for the required thickness, viz. 1·8 inches.

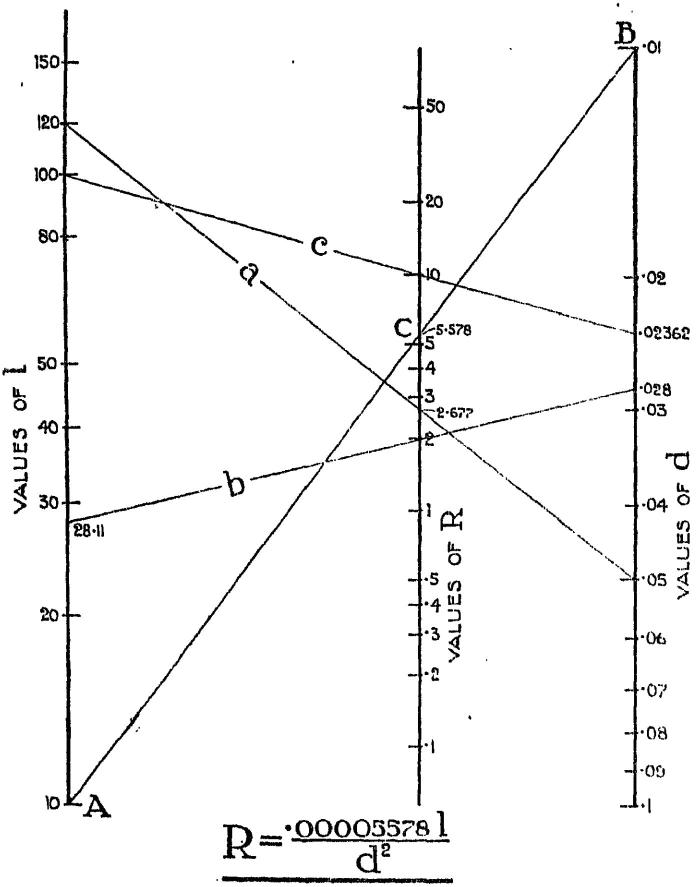


FIG. 36.—Resistance of Platinoid Wire.

Chart giving Resistance of Platinoid Wire

Example 3.—The electrical resistance R ohms of platinoid wire of diameter *d* cms. and length *l* cms. can be found from the formula

$$R = \frac{\cdot 00005578 l}{d^2}$$

Construct a chart to give the resistance if the length varies from 10 cms. to $1\frac{1}{2}$ metres, and the diameter varies from ·01 to ·1 cm.

Writing the equation in the log form

$$\log R = \log \cdot 00005578 + \log l - 2 \log d$$

or $$\log l - 2 \log d = \log R - \log \cdot 00005578$$

so that the axis of R may be placed between those of l and d, provided that the axis of d is graduated in the opposite direction from the axis of l.

Now l ranges from 10 to 150, $\log l$ ranges from 1 to 2·1761, *i. e.* a difference of about 1·2 units : let 4″ represent this range, so that $l_1 = \cdot 3$.

Also d varies from ·01 to ·1, $\log d$ from $\bar{2}$ to $\bar{1}$, and $2 \log d$ from $\bar{4}$ to $\bar{2}$, a range of 2 units : hence a suitable scale-value is $l_2 = \cdot 5$.

Then $l_3 = l_1 + l_2 = \cdot 3 + \cdot 5 = \cdot 8$ and $\dfrac{m_1}{m_2} = \dfrac{l_2}{l_1} = \dfrac{\cdot 5}{\cdot 3} = \dfrac{5}{3}$

so that, if $m = 3''$, $m_1 = \frac{5}{8} \times 3 = 1\frac{7}{8}''$

The axes of l and d are spaced 3″ apart, the graduations along the former being from A upward, and along the latter from B (which is any convenient point) downward.

For the axis of l, the distance from A for a marking 100, for example, $= \dfrac{\log 100 - \log 10}{\cdot 3} = 3\cdot 333''$ and the table of values for a few points is

l	10	20	30	40	50	80	100	120	150
$\log l$	1	1·3010	1·4771	1·6021	1·6990	1·9031	2	2·0792	2·1761
$\log l - \log 10$ . . .	0	·3010	·4771	·6021	·6990	·9031	1	1·0792	1·1761
Distance from A (inches)	0	1·003	1·59	2·007	2·33	3·010	3·333	3·597	3·921

For the axis of d, 1″ = ·5 unit of $2 \log d$ and the virtual zero is at $d = \cdot 01$, at the point B; hence the table reads

d	·01	·02	·03	·04	·05	·06	·07	·08	·09	·1
$\log d$	$\bar{2}$	$\bar{2}\cdot 301$	$\bar{2}\cdot 4771$	$\bar{2}\cdot 6021$	$\bar{2}\cdot 699$	$\bar{2}\cdot 7782$	$\bar{2}\cdot 8451$	$\bar{2}\cdot 9031$	$\bar{2}\cdot 9542$	$\bar{1}$
$2 \log d$	$\bar{4}$	$\bar{4}\cdot 602$	$\bar{4}\cdot 9542$	$\bar{3}\cdot 2042$	$\bar{3}\cdot 398$	$\bar{3}\cdot 5564$	$\bar{3}\cdot 6902$	$\bar{3}\cdot 8062$	$\bar{3}\cdot 9084$	$\bar{2}$
$2 (\log d - \log \cdot 01)$.	0	·602	·9542	1·2042	1·398	1·5564	1·6902	1·8062	1·9084	2
Distance from B (inches) . . .	0	1·204	1·9084	2·4084	2·796	3·1128	3·3804	3·6124	3·8168	4

The line AB cuts the axis of R at C, and the value of R to be indicated here is 5·578, since the required value of $R = \dfrac{\cdot 00005578 \times 10}{(\cdot 01)^2} = 5\cdot 578.$

Now $l_3 = \cdot 8$ and thus the table reads

R	5·578	1	2	3	4	5	10	20	50
log R. . . .	·7465	0	·3010	·4771	·6021	·699	1	1·301	1·699
log R − log 5·578	0	−·7465	−·4455	−·2694	−·1444	−·0475	·2535	·5545	·9525
Distance from C (inches) .	0	−·933	−·557	−·337	−·1805	−·0594	·317	·693	1·1906

To Use the Chart.—(*a*) Find the resistance of 120 cms. of platinoid wire of dia. ·05 cm. Join 120 on the scale of l to ·05 on the scale of d to cut the axis of R at the required resistance 2·677 ohms.

(*b*) What length of wire of dia. ·028 cm. has a resistance of 2 ohms? Join ·028 on the axis of d to 2 on the axis of R and produce the line to cut the axis of l at the required length, viz. 28·11 cms.

(*c*) To find the diameter of the wire, 1 metre of which has a resistance of 10 ohms. Join 100 on the axis of l to 10 on the axis of R and produce the line to cut the axis of d at ·02362.

The next Example gives a chart of the same form as that in Fig. 35, but the calculation necessary for its construction is rather more involved.

Chart for Lloyd's Rule for Furnace Tubes

Example 4.—Lloyd's rule for the thickness of Morrison's furnace tube is

$$P = \frac{1259(16t - 2)}{D}$$

where P = pressure of steam in lbs. per sq. inch
D = diameter (in inches) outside the corrugations
and t = thickness of metal (in inches).

Construct a chart to give values of the thickness when P varies from 20 to 200, and D varies from 10 to 50.

The equation may be written as

$$\frac{PD}{1259} = (16t - 2)$$

or

$$\frac{P}{20} \times \frac{D}{62\cdot 95} = (16t - 2)$$

which is evidently of the form $uv = w$.

Now P ranges from 20 to 200, $\frac{P}{20}$ from 1 to 10, and log $\frac{P}{20}$ from 0 to 1, so that a convenient scale is ·2 unit per inch, *i. e.* $l_1 = \cdot 2$. Also D ranges from 10 to 50, $\frac{D}{62\cdot95}$ from ·1589 to ·7945, and log $\frac{D}{62\cdot95}$ from $\bar{1}\cdot201$ to $\bar{1}\cdot9001$, a range of ·7 unit, so that again we may take the scale-value as ·2.

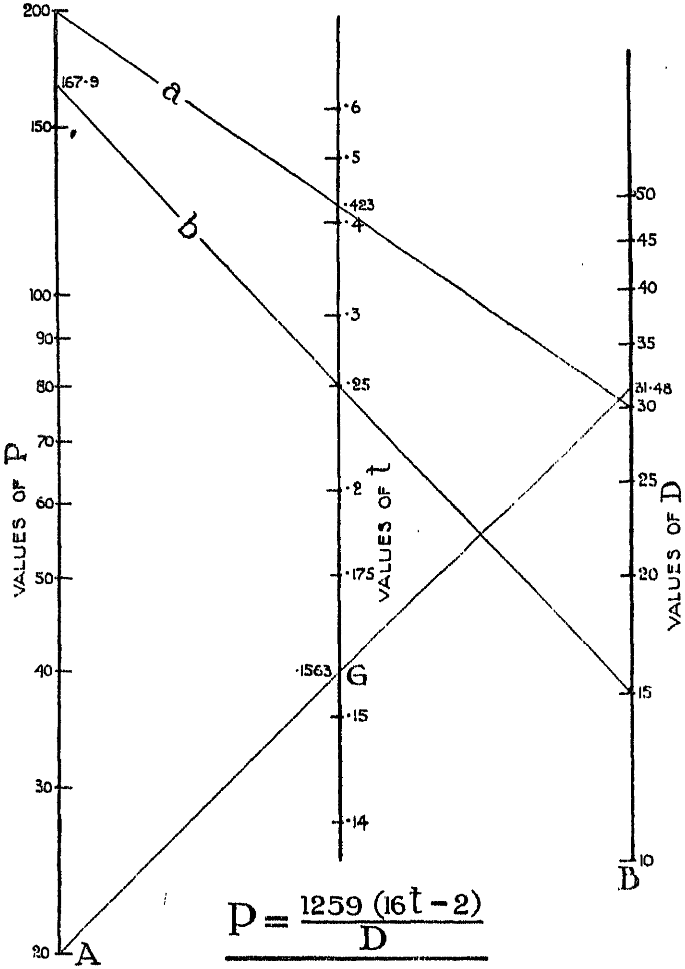


FIG. 37.—Lloyd's Rule for Furnace Tubes.

Thus $l_2 = \cdot 2$, $l_3 = l_1 + l_2 = \cdot 4$ and $m_1 = m_2 = \frac{1}{2}m$.

Drawing our axes for P and D, 3 inches apart (Fig. 37), we proceed to graduate them, using the following tables, it being only necessary to deal with log P and log D and not log $\frac{P}{20}$ and log $\frac{D}{62\cdot 95}$ for the reason explained in *Example* 2, p. 58.

For the axis of P, $l_1 = \cdot 2$ and P = 20 is the virtual zero, and the table is

P	20	30	40	50	60	70	80	90	100	150	200
log P . . .	1·301	1·4771	1·6021	1·699	1·7782	1·8451	1·9031	1·9542	2	2·1761	2·301
log P − log 20 .	0	·1761	·3011	·398	·4772	·5441	·6021	·6532	·699	·8751	1
Distance from A (inches) .	0	·881	1·506	1·99	2·386	2·721	3·011	3·266	3·495	4·376	5

For the axis of D, $l_2 = \cdot 2$ and the virtual zero is at D = 10; and the table is

D	10	15	20	25	30	35	40	45	50
log D	1	1·1761	1·301	1·3979	1·4771	1·5441	1·6021	1·6532	1·699
log D − log 10 . . .	0	·1761	·301	·3979	·4771	·5441	·6021	·6532	·699
Distance from B (inches) .	0	·881	1·505	1·99	2·386	2·721	3·011	3·266	3·495

A starting value for the t axis can now be found by a simple calculation :—

$$\frac{P}{20} \times \frac{D}{62\cdot 95} = (16t - 2).$$

If P = 20 and D = 31·48

$$(16t - 2) = 1 \times \cdot 5 = \cdot 5$$
$$16t = \cdot 5 + 2 = 2\cdot 5$$

and $$t = \cdot 1563.$$

Hence, if P = 20 be joined to D = 31·48, the point of intersection G with the axis of t must be marked ·1563 and then all distances can be calculated as from G; the table being set out in the following manner :—

t	·1563	·14	·15	·175	·2	·25	·3	·4	·5	·6
$16t - 2$	·5	·24	·40	·8	1·2	2	2·8	4·4	6	7·6
log $(16t - 2)$ . .	$\bar{1}\cdot 699$	$\bar{1}\cdot 3802$	$\bar{1}\cdot 6021$	$\bar{1}\cdot 9031$	·0792	·301	·4472	·6435	·7782	·8808
log $(16t-2)$ − log ·5	0	− ·3188	− ·0969	·2041	·3802	·602	·7482	·9445	1·0792	1·1818
Distance from G (inches) . .	0	− ·797	− ·242	·51	·951	1·505	1·871	2·361	2·698	2·955

To Use the Chart.—(*a*) Given that $P = 200$ and $D = 30$, to find t. Join 200 on the axis of P to 30 on the axis of D to cut the axis of t at the required value, viz. ·423.

(*b*) If $D = 15$, and $t = {\cdot}25$, to find P. Join $D = 15$ to $t = {\cdot}25$ and produce the line to cut the axis of P at $P = 167{\cdot}9$.

Chart giving Gross Discounts

Example 5.—Construct a chart to give gross discounts C which are dependent on two other discounts x and y, the relation being $x + y - {\cdot}01xy = C$, the values of x ranging from 1 to 100, and those of y from 0 to 50 (Fig. 38).

It is not possible to get a very simple chart to represent this equation as it stands, but if the equation is transposed a chart can be constructed which, while not completely satisfying the original equation, assists in its evaluation.

Thus
$$x + y - {\cdot}01xy = C$$
$$x(1 - {\cdot}01y) = C - y = K$$
and
$$\log x + \log(1 - {\cdot}01y) = \log K$$

Hence a chart may be constructed to give values of K, and from these values of K those of C may readily be found by adding the particular values of y.

Detailed description of the graduation of the axis of x is not necessary, since the scale is a simple logarithmic scale, with $l_1 = {\cdot}4$. For the y axis, it is best to make a table first; and the scale-value may be taken as ·1, since the range of values is from $\log(1 - {\cdot}01 \times 0)$, *i. e.* 0 to $\log(1 - {\cdot}01 \times 50)$, *i. e.* $\bar{1}{\cdot}699$, or ·301 unit.

Thus $l_1 = {\cdot}4$, $l_2 = {\cdot}1$, $l_3 = l_1 + l_2 = {\cdot}5$, and if $m = 3''$, $m_1 = \frac{1}{5} \times 3 = {\cdot}6''$. Then for the axis of y the tabulation is

y	0	5	10	15	20	25	30	35	40	45	50
$(1 - {\cdot}01y)$ . . .	1	·95	·9	·85	·8	·75	·7	·65	·6	·55	·5
$\log(1 - {\cdot}01y)$ . .	0	$\bar{1}{\cdot}9777$	$\bar{1}{\cdot}9542$	$\bar{1}{\cdot}9294$	$\bar{1}{\cdot}9031$	$\bar{1}{\cdot}8751$	$\bar{1}{\cdot}8451$	$\bar{1}{\cdot}8129$	$\bar{1}{\cdot}7782$	$\bar{1}{\cdot}7404$	$\bar{1}{\cdot}699$
$\log(1 - {\cdot}01y) - \log 1$	0	− ·0223	− ·0458	− ·0706	− ·0969	− ·1249	− ·1549	− ·1871	− ·2218	− ·2596	− ·301
Distance from A (downwards) .	0	·223	·458	·706	·969	1·249	1·549	1·871	2·218	2·596	3·01

The easiest way to graduate the axis of K is to join 0 on the axis of y to the various markings on the x axis. Thus if $y = 0$ is joined to $x = 3$, the intersection of this line with the axis of K must be marked 3, for $K = x(1 - {\cdot}01y) = 3(1 - 0) = 3$.

To Use the Chart.—(*a*) If $x = 20\%$ and $y = 10\%$, to find

C. Join $x = 20$ to $y = 10$ to cut the axis of K at 18: then $C = K + y = 18 + 10 = 28\%$.

(*b*) If $C = 30\%$ and $y = 5\%$, to find x. $K = C - y = 30 - 5 = 25$: hence join 5 on the y axis to 25 on the axis of K and produce the line to cut the x axis at the required value, viz. 26·31.

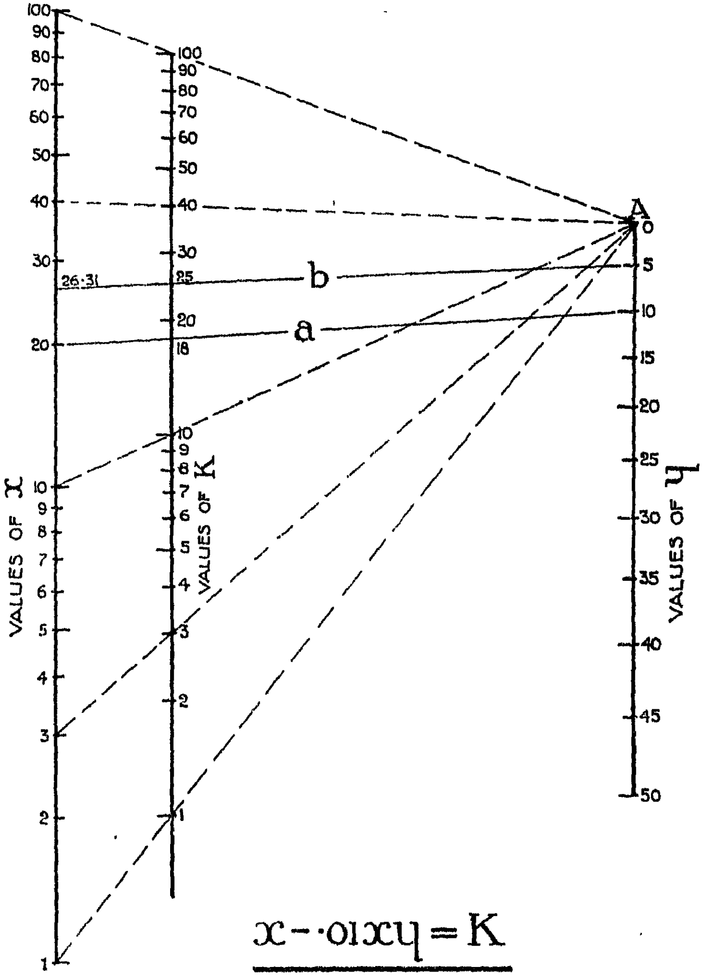


FIG. 38.

The sphere of usefulness of this type of chart can be enlarged by the inclusion of formulæ dependent upon the trigonometric functions. For charts of this type the general method of procedure is the same as that already described, but the functional scales are of the character indicated in Fig. 4.

Two examples of this type are introduced here, the first being worked by two methods so that a further comparison may be made between the alignment chart and the intercept chart. It will be seen that whenever awkward powers of the variables occur in the formulæ the calculation necessary in the construction of the intercept chart is not much less than that needed for the alignment chart, whilst the latter is considerably easier to use. The intercept chart is of value in such simple cases as detailed in *Example* 1, p. 54, but for general usage the alignment chart is to be preferred.

Charts giving Flow of Water over a Notch.

Example 6.—The quantity Q of water, in cu. ft. per sec., flowing over a triangular notch can be found from the formula

$$Q = \frac{8}{15} \tan \frac{\theta}{2} \sqrt{2g}\, H^{\frac{5}{2}}$$

where H = head at the notch (in feet).

Construct (A) an alignment chart, and (B) an intercept chart to give values of Q as θ varies from 30° to 90° and H varies from ·1 to 1·25.

(A)

$$Q = \frac{8}{15}\sqrt{2g} \tan \frac{\theta}{2} . H^{\frac{5}{2}}$$

$$= 4{\cdot}281 \tan \frac{\theta}{2} . H^{\frac{5}{2}}$$

or

$$\frac{Q}{4{\cdot}281} = \tan \frac{\theta}{2} . H^{\frac{5}{2}}$$

In the log form $\log Q - \log 4{\cdot}281 = \log \tan \frac{\theta}{2} + 2{\cdot}5 \log H.$

Then, θ varies from 30° to 90°, $\frac{\theta}{2}$ from 15° to 45° and $\log \tan \frac{\theta}{2}$ from $\bar{1}{\cdot}4281$ to 0, a range of about ·6 unit: hence l_1 may be taken as ·1. Also 2·5 log H varies from 2·5 log ·1 to 2·5 log 1·25, *i. e.* from $2{\cdot}5 \times \bar{1}$ to $2{\cdot}5 \times {\cdot}0969$, a range of about 2·7 units: hence l_2 may be taken as ·5.

Thus

$$l_3 = l_1 + l_2 = {\cdot}1 + {\cdot}5 = {\cdot}6 \quad \text{and} \quad \frac{m_1}{m_2} = \frac{{\cdot}5}{{\cdot}1} \quad \text{or} \quad m_1 = \frac{5}{6} \text{ of } m,$$

so that if $m = 3$ inches, the axis of Q is placed $2\frac{1}{2}''$ from the axis of θ. The tabulation is as follows :—

For the axis of θ, ($l_1 = \cdot 1$)

θ	30	40	50	60	70	80	90
$\frac{\theta}{2}$	15	20	25	30	35	40	45
log tan $\frac{\theta}{2}$	$\bar{1}$·4281	$\bar{1}$·5611	$\bar{1}$·6687	$\bar{1}$·7614	$\bar{1}$·8452	$\bar{1}$·9238	0
log tan $\frac{\theta}{2}$ − log tan 15°	0	·1330	·2406	·3333	·4171	·4957	·5719
Distance from A (inches)	0	1·33	2·406	3·333	4·171	4·957	5·719

For the axis of H, $l_2 = \cdot 5$ and the actual distance from B of a marking $H_1 = \frac{2 \cdot 5}{\cdot 5}(\log H_1 - \log \cdot 1) = 5(\log H_1 - \log \cdot 1)$, so that the table reads

H	·1	·2	·3	·4	·5	·8	1	1·25
log H	$\bar{1}$	$\bar{1}$·301	$\bar{1}$·4771	$\bar{1}$·6021	$\bar{1}$·699	$\bar{1}$·9031	0	·0969
log H − log ·1	0	·301	·4771	·6021	·699	·9031	1	1·0969
Distance from B (inches) . .	0	1·505	2·386	3·01	3·495	4·516	5	5·485

The line joining H = ·1 to $\theta = 90°$ (Fig. 39) cuts the axis of Q at the point C, and this point must be marked ·01354, since

$$Q = 4 \cdot 281 \tan 45° \, (\cdot 1)^{2 \cdot 5}$$
$$\log Q = \log 4 \cdot 281 + \log \tan 45° + 2 \cdot 5 \log \cdot 1$$
$$= \cdot 6315 + 0 - 2 \cdot 5 = \bar{2} \cdot 1315 = \log \cdot 01354$$

or

$$Q = \cdot 01354$$

Thus the axis of Q can now be graduated from C, using the scale-value $l_3 = \cdot 6$, the table being

Q	·01354	·02	·05	·1	·2	·3	·5	1	2	5
log Q . . .	$\bar{2}$·1315	$\bar{2}$·301	$\bar{2}$·699	$\bar{1}$	$\bar{1}$·301	$\bar{1}$·4771	$\bar{1}$·699	0	·301	·699
log Q − log ·01354	0	·1605	·5675	·8685	1·1695	1·3456	1·5675	1·8685	2·1695	2·5675
Distance from C (inches)	0	·283	·946	1·448	1·949	2·243	2·613	3·114	3·616	4·279

To Use the Chart.—(*a*) To find Q when $\theta = 60°$ and H = ·3. Join 60 on the axis of θ to ·3 on the axis of H and note the point of intersection, viz. ·1218, with the axis of Q.

(*b*) Given that $\theta = 50°$ and Q = 2, to find H. Join $\theta = 50$ to Q = 2 and produce the line to cut the axis of H at 1.

(*c*) Given that $\theta = 80°$ and H = 1·15, to find Q. Join $\theta = 80$ to H = 1·15 to cut the axis of Q at 5·095.

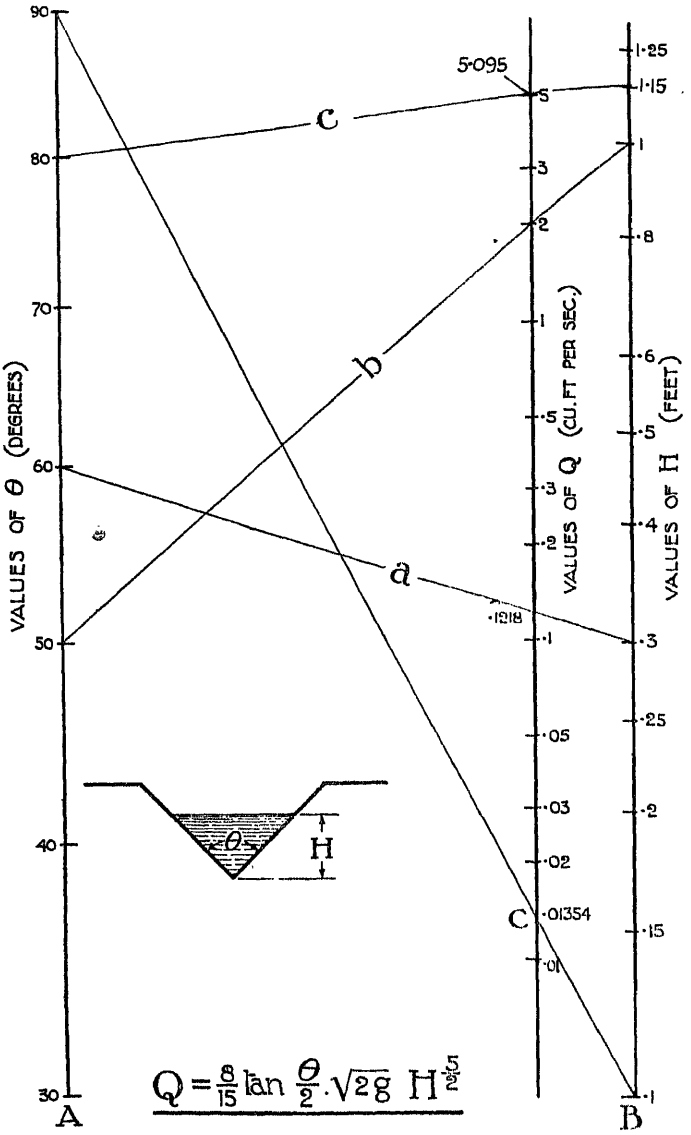


FIG. 39.—Flow of Water over a Notch.

(B) $$Q = 4{\cdot}281H^{\frac{5}{2}} \times \tan\frac{\theta}{2} = M \times \tan\frac{\theta}{2}$$

if M is written in place of $4{\cdot}281H^{\frac{5}{2}}$

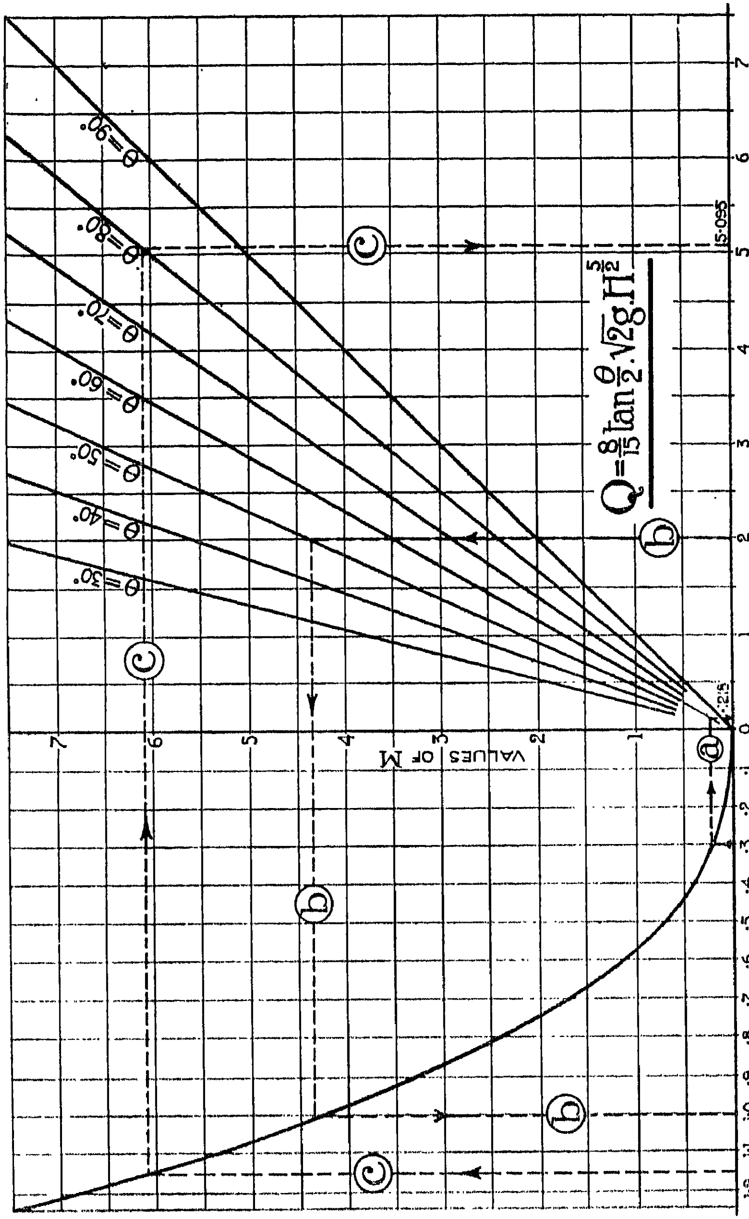


FIG. 40.—Flow of Water over a Notch.

so that a chart may be constructed of two distinct parts, that on the left-hand side being obtained by plotting M against H, and that on the right-hand side being a series of lines obtained by plotting Q against M for various values of θ.

For the left-hand chart (Fig. 40)

$$M = 4{\cdot}281H^{5/2}$$

whence $$\log M = \log 4{\cdot}281 + 2{\cdot}5 \log H$$

and a table of values is

H	log H	log 4·281+2·5 log H	log M	M
0	− ∞	·6315 − ∞	− ∞	0
·2	$\bar{1}$·301 = − ·699	·6315 − 1·7475	$\bar{2}$·8840	·07656
·4	$\bar{1}$·6021 = − ·3979	·6315 − ·9948	$\bar{1}$·6367	·4332
·6	$\bar{1}$·7782 = − ·2218	·6315 − ·5545	·0770	1·194
·8	$\bar{1}$·9031 = − ·0969	·6315 − ·2423	·3892	2·45
1·0	0	·6315 + 0	·6315	4·281
1·25	·0969	·6315 + ·2423	·8738	7·478

from which the curve shown in the diagram can be plotted. Now Q varies directly as M, and $Q = 0$ when $M = 0$, so that it is only necessary to calculate for one point on each line.

Thus, when $M = 5$ and $\theta = 70°$, for example,

$$Q = 5 \times \tan 35° = 3{\cdot}501,$$

so that the line joining the origin to the point (3·501, 5) must be indicated as that for $\theta = 70°$. In like manner other lines may be drawn.

To Use the Chart.—(*a*) To find Q when $\theta = 60°$ and $H = {\cdot}3$: erect a perpendicular at $H = {\cdot}3$ to meet the curve, then draw a horizontal line to intersect the line marked $\theta = 60°$, and project down to cut the axis of Q at the required value, viz. ·1218.

(*b*) Given that $\theta = 50°$ and $Q = 2$, to find H. Erect a perpendicular at $Q = 2$ to cut the line marked $\theta = 50°$, then draw a horizontal line to cut the curve, and project down to cut the axis of H at 1. Line (*c*) shows the determination of the value, 5·095, of Q when $\theta = 80°$ and $H = 1{\cdot}15$, the procedure being as detailed in (*a*).

Chart giving Wind Pressure on an Inclined Surface

Example 7.—Duchemin's rule for the normal intensity p_n of wind pressure on a surface inclined to the direction of the wind at an angle α is

$$p_n = p\frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

where p is the intensity of the horizontal wind pressure on a vertical surface. An approximate rule connecting p (in lbs. per sq. ft.) and v (the velocity of the wind, in miles per hour) is $p = \frac{v^2}{200}$. Construct a chart to give values of p_n as α varies from 15° to 60° and v varies from 10 to 100 miles per hour.

Combining the two formulæ given

$$p_n = \frac{v^2}{200} \times \frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

or

$$p_n = \frac{v^2}{100} \times \frac{\sin \alpha}{1 + \sin^2 \alpha}$$

and in the log form

$$\log p_n + \log 100 = 2 \log v + \log \left(\frac{\sin \alpha}{1 + \sin^2 \alpha} \right)$$

Now v ranges from 10 to 100 m.p.h. and thus $2 \log v$ ranges from 2×1 to 2×2, a range of 2 units, so that a suitable scale-value is $l_1 = \cdot 5$.

$$\text{When} \quad \alpha = 15^\circ \quad \frac{\sin \alpha}{1 + \sin^2 \alpha} = \frac{\cdot 2588}{1 + (\cdot 2588)^2} = \cdot 2426$$

$$\text{and } \log \left(\frac{\sin \alpha}{1 + \sin^2 \alpha} \right) = \bar{1}\cdot 3849$$

$$\text{and when} \quad \alpha = 60^\circ \quad \frac{\sin \alpha}{1 + \sin^2 \alpha} = \frac{\cdot 866}{1 + (\cdot 866)^2} = \cdot 4949$$

$$\text{and } \log \left(\frac{\sin \alpha}{1 + \sin^2 \alpha} \right) = \bar{1}\cdot 6945$$

so that the scale must show a difference of ·3096; a suitable scale-value being $l_2 = \cdot 1$.

Then $l_3 = l_1 + l_2 = \cdot 6$ and $\frac{m_1}{m_2} = \frac{\cdot 1}{\cdot 5}$ or $m_1 = \frac{1}{6}$ of m. In Fig. 41 $m = 3''$ and hence $m_1 = \frac{1}{2}''$. The scale of v can be set out without any difficulty, but for the axis of α the calculation necessary must be indicated, thus :—

α	$\sin \alpha$	$\sin^2 \alpha$	$\frac{\sin \alpha}{1 + \sin^2 \alpha}$	$\log \left(\frac{\sin \alpha}{1 + \sin^2 \alpha} \right)$	Difference of logs	Distance from A
15°	·2588	·067	·2426	$\bar{1}\cdot3849$	0	0
20	·342	·117	·3062	$\bar{1}\cdot4860$	·1011	1·011
30	·5	·25	·4	$\bar{1}\cdot6021$	·2172	2·172
40	·6428	·4132	·4548	$\bar{1}\cdot6579$	·2730	2·73
50	·766	·5868	·4826	$\bar{1}\cdot6836$	·2987	2·987
60	·866	·75	·4949	$\bar{1}\cdot6945$	·3096	3·096

The intersection at B with the axis of p_n of the line joining $v = 80$ to $\alpha = 60°$ gives a starting value, viz. 31·67, for the graduation of this axis for

$$p_n = \frac{v^2}{100} \times \frac{\sin \alpha}{1 + \sin^2 \alpha} = \frac{6400}{100} \times \cdot 4949 = 31 \cdot 67$$

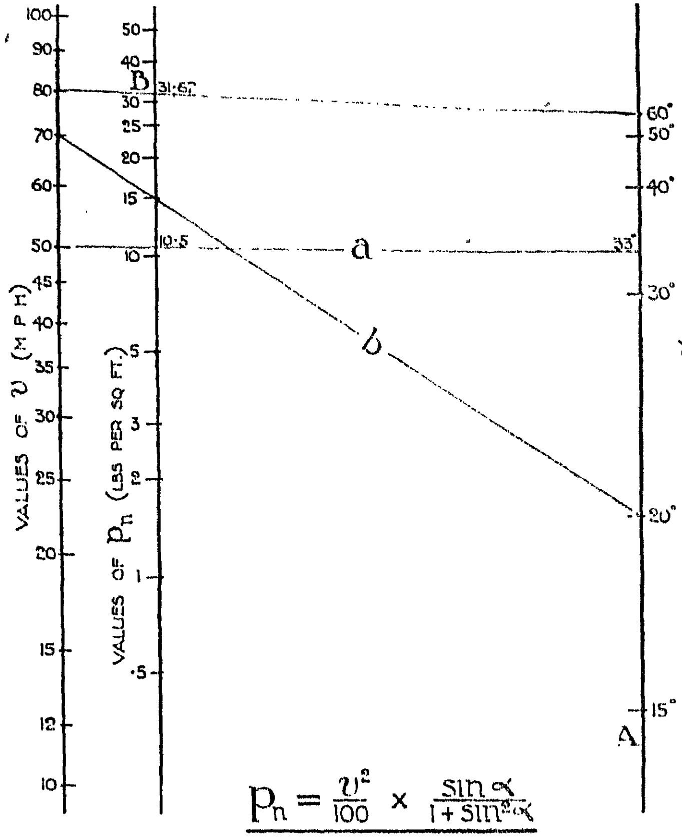


FIG. 41.—Pressure of Wind on an Inclined Surface.

and all the distances must be measured downward; the table being

p_n	31·67	30	25	20	15	10	5	1
log p_n	1·5007	1·4771	1·3979	1·301	1·1761	1	·699	0
log 31·67 − log p_n . . .	0	·0236	·1028	·1997	·3246	·5007	·8017	1·5007
Distance from B	0	·039	·171	·333	·541	·835	1·336	2·501

It should be noticed that for all logarithmic scales the distances between the markings for numbers having the same relation to one another are the same: thus the distance from 1 to 2 is the same as that from 10 to 20, 20 to 40, 15 to 30, etc., so that the markings for 40, 50, 45, etc., can readily be fixed from the figures in the table.

To Use the Chart.—(*a*) If the wind velocity is 50 m.p.h., and the pitch of a roof is 33°, what is the normal intensity of the wind pressure on the roof? Join 50 on the axis of v to 33 on the axis of α to cut the axis of p_n at 10·5: then the intensity is 10·5 lbs. per sq. foot.

(*b*) To find the wind velocity when the normal wind pressure on a roof inclined at 20° to the horizontal is 15 lbs. per sq. foot. Join 20 on the axis of α to 15 on the axis of p_n and produce the line to cut the axis of v at the required speed, viz. 70 m.p.h.

The advantage of a chart is clearly seen in this example, for the calculation is somewhat involved, whereas the solution by the chart is direct.

The next Example is illustrative of the method of dealing with powers and roots, and again demonstrates the great usefulness of the solution by a chart.

Chart giving Resistance of a Moving Plane

Example 8.—The formula given by the N.P.L. for the skin resistance on a plane is

$$R = \cdot 0000082\ A^{\cdot 93}V^{1\cdot 86}$$

where R = skin resistance in lbs. per sq. ft.
A = area in sq. ft.
and V = speed in ft. per sec.

If A varies from 1 to 30 and V from 20 to 120, construct a chart to give values of R (Fig. 42).

Transposing the formula $\dfrac{R}{\cdot 0000082} = A^{\cdot 93}V^{1\cdot 86}$

In the log form $\log R - \log \cdot 0000082 = \cdot 93 \log A + 1\cdot 86 \log V$.

For the axis of A the range is from ·93 log 1 to ·93 log 30, *i. e.* from 0 to 1·3737, and a convenient scale-value is $l_1 = \cdot 31$, this particular value being chosen since ·93 is exactly divisible by it. Thus the distance of the marking for a value A_1 from the marking for A = 1 is $\dfrac{\cdot 93}{\cdot 31}(\log A_1 - \log 1)$, *i. e.* $3 \log A_1$ inches.

For the axis of V the range is from 1·86 log 20 to 1·86 log 120, *i. e.* a range of 1·4475 units, and again we may take ·31 as the scale-value. Then $l_3 = l_1 + l_2 = ·62$ and $m_1 = m_2 = \frac{1}{2}m = 1\frac{1}{2}$ inches, so that the axis of R is placed mid-way between the axes of A and V.

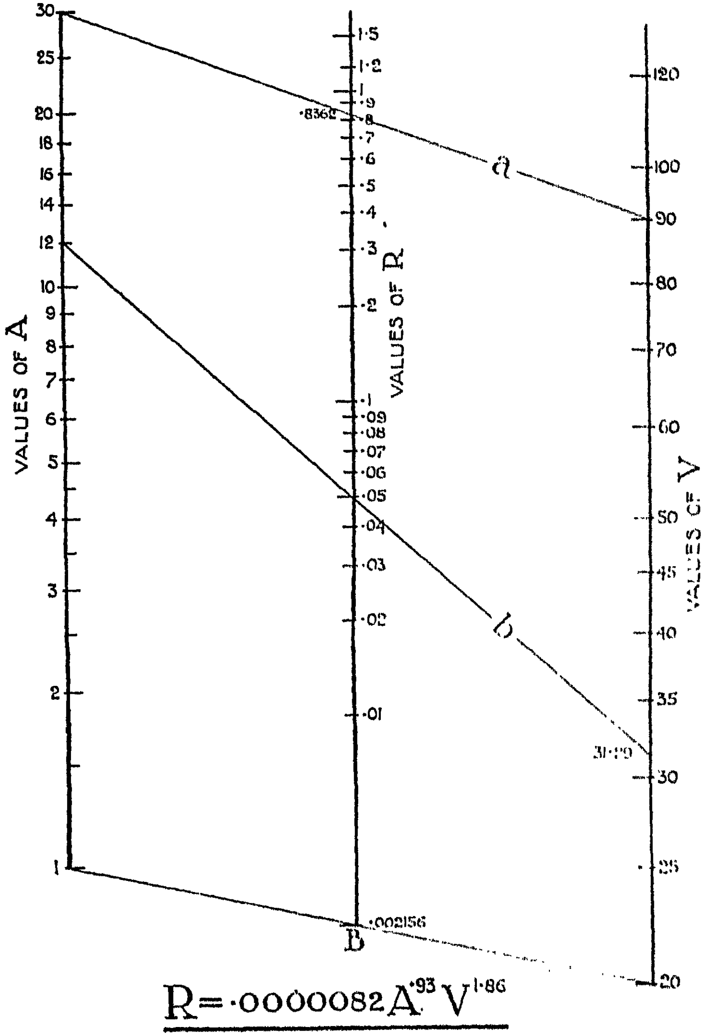


FIG. 42.—Resistance of a Moving Plane.

A convenient starting point on the R axis is found by joining A = 1 to V = 20, and marking the value as ·002156,

since
$$\begin{aligned} \log R &= \log \cdot 0000082 + \cdot 93 \log 1 + 1\cdot 86 \log 20 \\ &= \bar{6}\cdot 9138 + 0 + 1\cdot 86 \times 1\cdot 301 = \bar{3}\cdot 3337 \end{aligned}$$
whence
$$R = \cdot 002156$$

The graduation of the axis of R may be effected by either of two methods, viz. :—

(*a*) By finding the distance of the marking for ·01 from that for ·002156 $\left[\text{this being } \frac{1}{\cdot 62}(\log \cdot 01 - \log \cdot 002156) \text{ inches}\right]$ and then using a scale with scale-value ·62, such as could be obtained from a chart of the form shown in Fig. 32, calling the markings ·01, ·02 . . . ·1 instead of 1, 2 . . . 10, or

(*b*) By compiling a table in which distances from the marking for ·002156 are shown, a few values being

R	·002156	·01	·05	·1	·5	1	1·2	1·5
log R	$\bar{3}\cdot 3337$	$\bar{2}$	$\bar{2}\cdot 699$	$\bar{1}$	$\bar{1}\cdot 699$	0	·0792	·1761
log R − log ·002156 . .	0	·6663	1·3653	1·6663	2·3653	2·6663	2·7455	2·8424
Distance from B (inches) .	0	1·075	2·202	2·688	3·815	4·3	4·428	4·585

Line (*a*) shows the finding of R = ·8362 when A = 30 and V = 90; and by line (*b*) the value of V, 31·29, is determined when A = 12 and R = ·05, the procedure being apparent.

In order to show the utility of the chart the second of these cases is treated arithmetically :—

$$\begin{aligned} R &= \cdot 0000082\, A^{\cdot 93} V^{1\cdot 86} \\ 1\cdot 86 \log V &= \log R - \log \cdot 0000082 - \cdot 93 \log A \\ &= \log \cdot 05 - \bar{6}\cdot 9138 - \cdot 93 \log 12 \\ &= \bar{2}\cdot 699 - \bar{6}\cdot 9138 - 1\cdot 0037 = 2\cdot 7815 \\ \log V &= 1\cdot 4954 \\ V &= 31\cdot 29 \end{aligned}$$

Equations containing more than three variable quantities may be charted by a simple extension of the method used when there are the three variables only, and it is in such cases that the superiority of the alignment chart over the ordinary intercept graph is more pronounced. As the number of variables increases, however, the "practical" difficulties of the chart construction are more evident : thus the choice of scales and the relative placing of the various axes on the paper (particularly in the vertical direction) are factors needing careful thought and experience, if the chart is to be of the greatest efficiency. To fully explain the procedure in the case of an equation with four variables, two examples are here worked in detail, the first being of a rather elementary character.

Chart correlating Four Variables

Example 9.—Construct a chart to correlate the variables in the equation $uv = \cdot24pq$, the values varying as follows:—

u from 1 to 10
v from ·3 to 25
and p from 500 to 2500.

This chart (Fig. 43) is made up of two parts, linked together by a common mid-vertical; the two charts representing the equations $uv = w$ and $w = \cdot24pq$ respectively. It is not necessary, however, to graduate the axis of w.

Commence by drawing the vertical axes of u and v, 3 inches apart, and graduating them logarithmically with the scale-values l_1 (for the u axis) $= \cdot25$ and l_2 (for the v axis) $= \cdot5$. Then the mid-vertical must be so placed between them that $\frac{m_1}{m_2} = \frac{l_2}{l_1} = \frac{\cdot5}{\cdot25} = 2$, or $m_1 = 2''$ and $m_2 = 1''$, and l_3, the scale-value for this mid-vertical is $\cdot5 + \cdot25$, *i. e.* ·75. By joining $u = 8$ to $v = 3$, the point G is obtained, the value of w at this point being 24. This is the only point along the axis of w for which the value of w need be known.

Now the values of p must range from 500 to 2500, and $\log p$ varies from 2·699 to 3·3979, a range of ·6989 unit; so that a scale-value of $l_4 = \cdot2$ is suitable. Knowing that $l_4 = \cdot2$ and $l_3 = \cdot75$, we find that l_5 must be $\cdot75 - \cdot2$, *i. e.* ·55, since the axis of w serves as the mid-vertical between the axes of p and q. Then $\frac{m_4}{m_5} = \frac{l_5}{l_4} = \frac{\cdot55}{\cdot2} = 2\cdot75$, so that if the axis of q is drawn 1·6 inches to the right of the mid-vertical, the axis of p must be distant $2\cdot75 \times 1\cdot6$ or 4·4 inches to the left. Often it is difficult to fix a convenient position for the starting value, and it may be advisable to lay out the graduations for the outside axes on separate strips of paper and to then move the strips up or down the axes until the most useful placings are found. Choosing a position for the marking of 500 on the axis of p, as indicated on the diagram, the axis is graduated with $l_4 = \cdot2$. If now we join 1000 on the axis of p to G and produce the line to cut the axis of q, we determine the position of the marking for ·1, for the value of w at G is 24 and hence

$$24 = \cdot24 \times 1000 \times q, \quad \text{or } q = \cdot1.$$

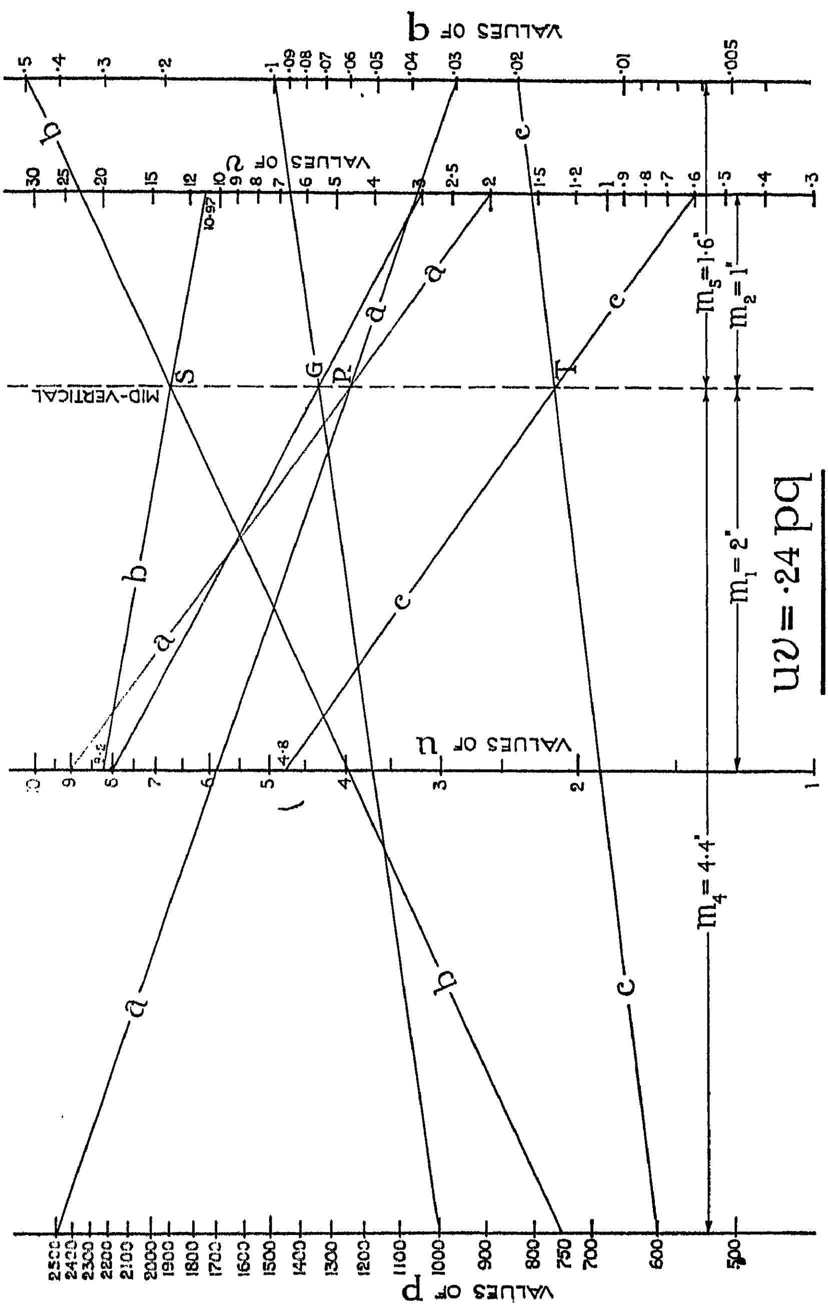

VALUES OF q
·5
·4
·3
·2
·1
·09
·08
·07
·06
·05
·04
·03
·02
·01
·005
VALUES OF v
30
25
20
15
12
10
9
8
7
6
5
4
3
2·5
2
1·5
1·2
1
·9
·8
·7
·6
·5
·4
·3
10·97
MID-VERTICAL
S
G
P
T
a
b
c
$m_5 = 1{\cdot}6''$
$m_2 = 1''$
$m_1 = 2''$
$m_4 = 4{\cdot}4''$
VALUES OF u
10
9
8·2
8
7
6
5
4·8
4
3
2
1
VALUES OF p
2500
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
750
700
600
500
$uv = {\cdot}24\,pq$

FIG. 43.

The axis of q can now be graduated using the scale-value $l_5 = \cdot 55$. It should be noted that the position of G must be found most carefully, for any error here has the effect of altering the correct relative placing of the q graduations. The chart is now complete and three examples of its use are shown.

(*a*) 'If $u = 9$, $v = 2$, and $p = 2500$, to find q. Join $u = 9$ to $v = 2$ to cut the mid-vertical at R: join 2500 on the axis of p to R and produce to cut the axis of q at the required value ·03.

(*b*) Given that $q = \cdot 5$, $u = 8 \cdot 2$, and $p = 750$, to find v. Join $p = 750$ to $q = \cdot 5$ to cut the mid-vertical at S, and through S draw a line from 8·2 on the axis of u to cut the axis of v at 10·97.

(*c*) If $p = 600$, $q = \cdot 02$, and $v = \cdot 6$, to find u. Join $p = 600$ to $q = \cdot 02$ to cut the mid-vertical at T. Join $v = \cdot 6$ to T and produce to cut the axis of u at $u = 4 \cdot 8$.

It must be noted that the chart is constructed by associating u with v and p with q and that this grouping must be adhered to in the use of the chart: thus in case (*a*), for example, it would be incorrect to commence by joining $v = 2$ to $p = 2500$.

The next Example shows a chart of the same kind as in the preceding example, but the formula that it illustrates is more complex and the necessary calculation is correspondingly increased.

Chart giving Working Pressure for Steel Flues

Example 10.—Unwin's formula for the safe working pressure in cylindrical welded steel flues, with longitudinal lap-joint, is

$$P = 1472600 \frac{t^{2 \cdot 1}}{l^{\cdot 9} d^{1 \cdot 16}}$$

where t = thickness of plate (inches)
l = length of flue between strengthening rings (inches)
d = dia. of flue (inches)
and P = safe working pressure in lbs. per sq. in.

Construct a chart to give the values of the variables for the ranges

t from ·1 to ·6
l ,, 10 to 70
and d ,, 10 to 50

The variables may be taken together in pairs, a suitable transposition of the factors giving

$$\frac{P l^{\cdot 9}}{1472600} = \frac{t^{2 \cdot 1}}{d^{1 \cdot 16}}$$

so that a chart is first constructed for $\frac{t^{2\cdot1}}{d^{1\cdot16}} = \text{K}$ and then one is built up from it for $\text{K} = \frac{Pl^{\cdot9}}{1472600}$.

Dealing first with the equation $\frac{t^{2\cdot1}}{d^{1\cdot16}} = \text{K}$, the range of t is from ·1 to ·6, so that $2\cdot1 \log t$ varies from $2\cdot1 \times \bar{1}$ to $2\cdot1 \times \bar{1}\cdot7782$, *i. e.* 1·6342 units, and $1\cdot16 \log d$ varies from $1\cdot16 \times 1$ to $1\cdot16 \times 1\cdot699$, a range of ·8108 unit: hence suitable scale-values are $l_1 = \cdot4$ for the axis of t, and $l_2 = \cdot2$ for the axis of d.

$$\text{Now} \qquad \log \text{K} = 2\cdot1 \log t - 1\cdot16 \log d$$

so that the axis of K can be placed between those of t and d, if the axis of d is graduated in the opposite direction from that of the axis of t (Fig. 44). Also $\frac{m_1}{m_2} = \frac{l_2}{l_1} = \frac{\cdot2}{\cdot4}$ so that $m_1 = \frac{1}{3}m$, and if m_1 is made ·7″, m_2 must = 1·4″.

The axes of t and d are now graduated, the line AB being the "virtual zero" line; and tables for the graduation of these axes are given.

For the axis of t

t	·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6
$\log t$	$\bar{1}$	$\bar{1}$·1761	$\bar{1}$·301	$\bar{1}$·3979	$\bar{1}$·4771	$\bar{1}$·5441	$\bar{1}$·6021	$\bar{1}$·6532	$\bar{1}$·699	$\bar{1}$·7404	$\bar{1}$·7782
$\log t - \log \cdot1$ . .	0	·1761	·301	·3979	·4771	·5441	·6021	·6532	·699	·7404	·7782
Distance from A (inches) . .	0	·9245	1·58	2·089	2·505	2·857	3·161	3·429	3·67	3·887	4·086

the values in the fourth line being obtained from those in the third line by multiplying by $\frac{2\cdot1}{\cdot4}$, *i. e.* by 5·25.

For the axis of d

d	10	15	20	25	30	35	40	45	50
$\log d$	1	1·1761	1·301	1·3979	1·4771	1·5441	1·6021	1·6532	1·699
$\log d - \log 10$	0	·1761	·301	·3979	·4771	·5441	·6021	·6532	·699
Distance from B (inches) .	0	1·021	1·746	2·308	2·767	3·156	3·492	3·789	4·054

the multiplying factor, giving the fourth line from the third line, being $\frac{1\cdot16}{\cdot2}$, *i. e.* 5·8.

The line AB intersects the mid-vertical at C: there is no need to graduate the mid-vertical, but its scale-value l_3 must be known, viz. $l_1 + l_2$, *i. e.* ·6, and also the value of K at C.

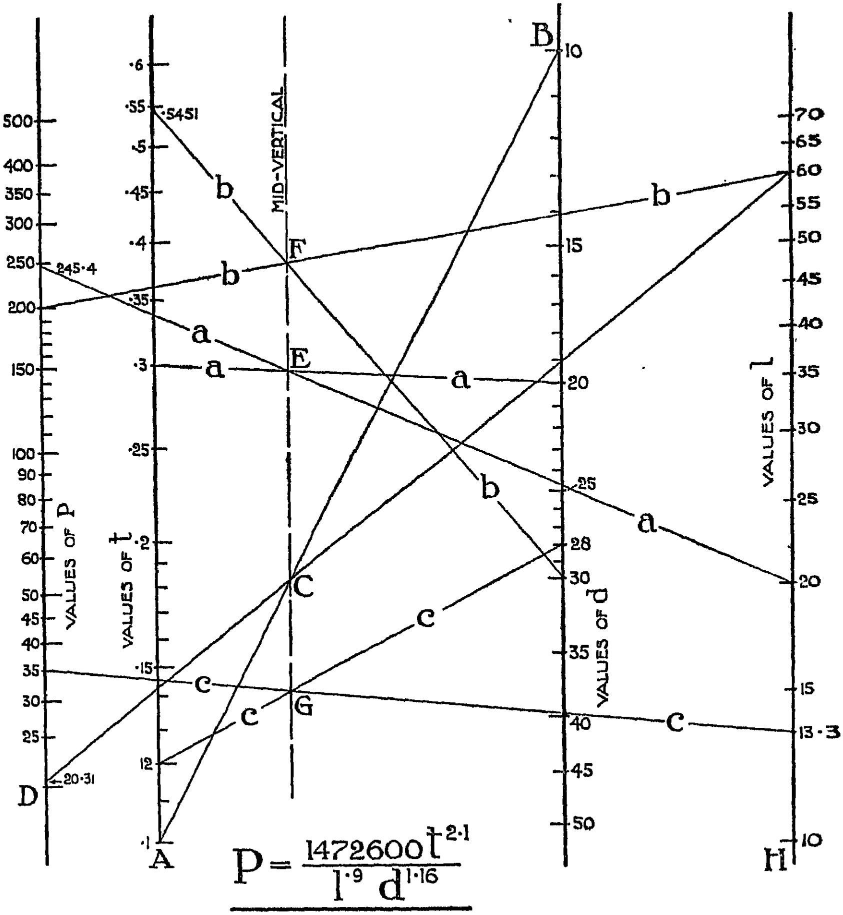


FIG. 44.—Unwin's Formula for Welded Steel Flues.

If $\quad K = \frac{t^{2\cdot1}}{d^{1\cdot16}}$

then, when $t = \cdot1$ and $d = 10$

$$\log K = 2\cdot1 \log \cdot1 - 1\cdot16 \log 10 = -2\cdot1 - 1\cdot16 = -3\cdot26 = \bar{4}\cdot74$$

and $\quad K = \cdot0005495$.

The range of l is from 10 to 70 and that of $\cdot 9 \log l$ is from $\cdot 9 \times 1$ to $\cdot 9 \times 1\cdot 8451$, *i. e.* $\cdot 7606$ unit. The scale-value l_4 for the axis of l must be considered in conjunction with l_3 and l_5, the scale-value for the axis of P.

If l_4 is taken as $\cdot 2$ then $l_5 = l_3 - l_4 = \cdot 4$, these scale-values being suitable for the axes concerned. For the axis of l a few values are indicated in the table :—

l	10	20	30	40	50	60	70
$\log l$	1	1·301	1·4771	1·6021	1·699	1·7782	1·8451
·9(log l — log 10)	0	·2709	·4294	·5419	·6291	·7004	·7606
Distance from H (inches) . .	0	1·3545	2·147	2·7095	3·1455	3·502	3·803

Since $l_4 = \cdot 2$ and $l_5 = \cdot 4$, $\dfrac{m_5}{m_4} = \dfrac{l_4}{l_5} = \dfrac{\cdot 2}{\cdot 4} = \dfrac{1}{2}$.

Hence, if the axis of l is distant 2·6 inches from the mid-vertical, the axis of P must be placed $\frac{1}{2} \times 2\cdot 6$ or 1·3 inches away from the mid-vertical. To fix a starting point for the graduation of the axis of P, we join 60 on the axis of l to C and produce the line to cut the axis of P at D, marking the value 20·31 at D; for

$$\frac{Pl^{\cdot 9}}{1472600} = K$$

$$P = \frac{1472600K}{l^{\cdot 9}} \qquad \text{and also } K = \cdot 0005495$$

$$\begin{aligned} \text{hence} \quad \log P &= \log 1472600 + \log \cdot 0005495 - \cdot 9 \log 60 \\ &= 6\cdot 1681 + \bar{4}\cdot 74 - \cdot 9 \times 1\cdot 7782 \\ &= 1\cdot 3077 \\ \text{or} \quad P &= 20\cdot 31 \end{aligned}$$

The graduation of the axis of P is most easily effected by working from the marking for 100 rather than that for 20·31, since in finding the differences between the logs it is much simpler to subtract 2 than to subtract 1·3077. The distance of the marking for 100 from D is readily found, it being $\dfrac{\log 100 - \log 20\cdot 31}{\cdot 4}$, *i. e.* $\dfrac{\cdot 6923}{\cdot 4}$, or 1·733 inches; and thence the axis can be graduated.

To Use the Chart.—(*a*) Given that $t = \cdot 3$, $d = 20$ and $l = 20$, to find P. Join $t = \cdot 3$ to $d = 20$, to cut the mid-vertical at E: join $l = 20$ to E and produce the line to cut the axis of P at 245·4.

Proof :—
$$
\begin{aligned}
\mathrm{P} &= 1472600 \times \frac{\cdot 3^{2\cdot 1}}{20^{\cdot 9} \times 20^{1\cdot 16}} \\
\log \mathrm{P} &= \log 1472600 + 2\cdot 1 \log \cdot 3 \\
&\qquad - (\cdot 9 \log 20 + 1\cdot 16 \log 20) \\
&= 6\cdot 1681 - 2\cdot 1 + 1\cdot 0019 - 1\cdot 1709 - 1\cdot 5092 \\
&= 2\cdot 3899 \\
\text{or} \qquad \mathrm{P} &= 245\cdot 4.
\end{aligned}
$$

(*b*) If $\mathrm{P} = 200$, $l = 60$ and $d = 30$, to find t. Join $\mathrm{P} = 200$ to $l = 60$ to cut the mid-vertical at F: join $d = 30$ to F, and produce the line to cut the axis of t at ·5451.

Proof :—
$$
\begin{aligned}
t^{2\cdot 1} &= \frac{\mathrm{P} l^{\cdot 9} d^{1\cdot 16}}{1472600} \\
2\cdot 1 \log t &= \log 200 + \cdot 9 \log 60 + 1\cdot 16 \log 30 - 6\cdot 1681 \\
&= 2\cdot 301 + 1\cdot 6004 + 1\cdot 7134 - 6\cdot 1681 \\
&= \bar{1}\cdot 4467 = -\cdot 5533 \\
\log t &= -\cdot 2635 = \bar{1}\cdot 7365 \\
\text{or} \qquad t &= \cdot 5451.
\end{aligned}
$$

(*c*) To find l when $\mathrm{P} = 35$, $t = \cdot 12$ and $d = 28$. Join $t = \cdot 12$ to $d = 28$ to cut the mid-vertical at G: join $\mathrm{P} = 35$ to G and produce the line to cut the axis of l at 13·3.

Proof :—
$$
\begin{aligned}
l^{\cdot 9} &= \frac{1472600 t^{2\cdot 1}}{\mathrm{P} d^{1\cdot 16}} \\
\cdot 9 \log l &= 6\cdot 1681 + 2\cdot 1 \log \cdot 12 - \log 35 - 1\cdot 16 \log 28 \\
&= 6\cdot 1681 - 2\cdot 1 + \cdot 1663 - 1\cdot 5441 - 1\cdot 6788 \\
&= 1\cdot 0115 \\
\log l &= 1\cdot 1239 \quad \text{or} \quad l = 13\cdot 3
\end{aligned}
$$

Observation of the amount of calculation necessary for the evaluation in the three cases considered shows the great value of the chart.

The next Example is of a chart of the same general character as those just discussed, but illustrates the duplication of the scale of one of the variables.

Chart for Horse-power transmitted by a Cotton Rope

Example 11.—Construct a chart to give the horse-power transmitted by a cotton rope of diameter d inches running at v feet per sec., using the formula

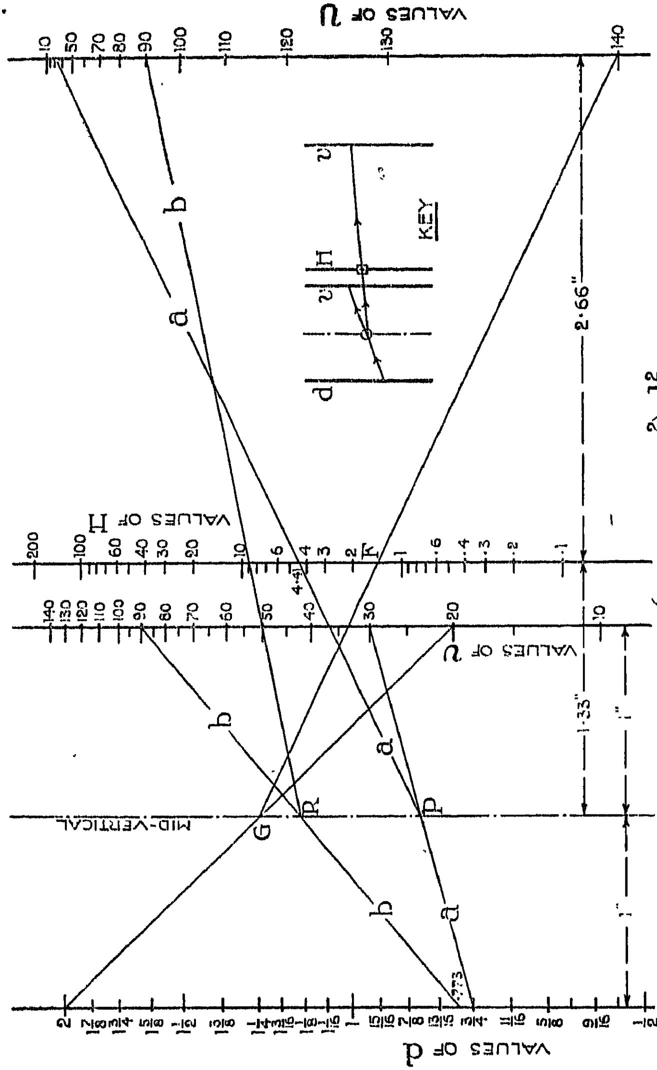
$$\mathrm{H} = \frac{(62800 - 3v^2)d^2 v}{230000}$$

v varying from 10 to 140, and d from $\frac{1}{2}$ to 2.

The formula can be expressed as

$$
\begin{aligned}
230000\,\mathrm{H} &= d^2 v \times (62800 - 3v^2) \\
&= \mathrm{N} \times \mathrm{M}
\end{aligned}
$$

so that a chart can be built up by relating N with d and v, and then relating H and M (which is a function of v) with N:



$$H = \frac{(62800 - 3v^2)d^2v}{230000}$$

FIG. 45.—Horse-power transmitted by a Cotton Rope.

the function N thus serving as a link, and the axis of N being merely an un-graduated mid-vertical.

Dealing first with the equation $N = d^2v$, we have

$\log N = 2 \log d + \log v$. Now d ranges from $\frac{1}{2}$ to 2, so that the range of $2 \log d$ is $2 \log 4$, *i. e.* 1·2042 units, and hence a scale-value of 1″ = ·4 unit is suitable: also $\log v$ varies from log 140 to log 10, *i. e.* from 2·1461 to 1, a range of 1·1461 units, so that the same scale-value will serve: thus $l_1 = l_2 = {\cdot}4$ and consequently l_3, the scale-value for the axis of N, is ·8 and the axis of N is equidistant from the axes of d and v. In Fig. 45 $m_1 = m_2 = 1$ inch.

The axes of d and v can be graduated, that of d being marked in fractions instead of the more usual decimals: the distances $\frac{1}{2}$ to $\frac{9}{16}$, $\frac{5}{8}$, $\frac{11}{16}$, etc., being of the nature $\frac{2 \log 1{\cdot}125}{{\cdot}4}$, $5 \log 1{\cdot}25$, $5 \log 1{\cdot}375$, and so on. By joining $d = 2$ to $v = 20$ a point G is found on the axis of N, the value at this point being $2^2 \times 20$ or 80.

We must now proceed to graduate the axis of M, the values along which are again values for v.

For the axis of M the range of values is $\log (62800 - 3 \times 10^2) - \log (62800 - 3 \times 140^2)$, *i. e.* from log 62500 to log 4000 or 1·1938 units, and a convenient scale-value is $l_4 = {\cdot}4$.

The table for the graduation of the axis of M, which is in reality a second axis of v, is

v	10	30	50	60	70	80	100	110	120	130	140
v^2	100	900	2500	3600	4900	6400	10000	12100	14400	16900	19600
$62800 - 3v^2$ . .	62500	60100	55300	52000	48100	43600	32800	26500	19600	12100	4000
$\log (62800 - 3v^2)$	4·7959	4·7789	4·7427	4·7160	4·6821	4·6395	4·5159	4·4232	4·2923	4·0828	3·6021
Difference of logs	0	·017	·0532	·0799	·1138	·1564	·28	·3727	·5036	·7131	1·1938
Distance from the marking for 10 . .	0	·043	·133	·2	·285	·391	·7	·932	1·259	1·783	2·985

the distances being measured in a downward direction.

The axis of M is drawn 4″ to the right of the mid-vertical, and then the position of the axis of H can be determined from a knowledge of the scale-values. Now $230000\,H = N \times M$ so that the axis of H must be placed between those of N and M, and since the scale-value l_3 for the axis of N is ·8, and l_4, for the axis of M is ·4, $\frac{m_3}{m_4} = \frac{l_4}{l_3} = \frac{{\cdot}4}{{\cdot}8} = \frac{1}{2}$ or the axis of H is distant $\frac{1}{3} \times 4$, *i. e.* 1·33″ from the axis of N (the mid-vertical).

The scale-value l_5 for the axis of H is $l_3 + l_4 = 1{\cdot}2$, and a starting point for the graduations is found by joining G, on the mid-vertical, to $v = 140$ on the axis of M. The value of H at this point F is found from $230000\,H = 80 \times 4000$, *i. e.*

H = 1·391, and it is the best plan to find the distance of the marking for 1 from F. This distance is $\frac{\log 1{\cdot}391 - \log 1}{1{\cdot}2}$, *i. e.* $\frac{{\cdot}1434}{1{\cdot}2}$ or ·119 inch, and thence the scale can readily be set out, using the scale-value 1·2.

To Use the Chart.—(*a*) To find H when $d = \frac{3}{4}''$ and $v = 30$. Join $d = \frac{3}{4}$ to $v = 30$ (on the left-hand v axis), to cut the mid-vertical at P. From P draw a line to 30 (on the right-hand v axis) to intersect the axis of H at the required value, viz. 4·41.

(*b*) Given that $v = 90$ and $H = 9$, to find d. Join 90 on the right-hand axis of v to 9 on the axis of H, and produce the line to cut the mid-vertical at R. Draw a line from 90 on the left-hand axis of v to R, and produce it to cut the axis of d at ·773.

Note that the left-hand axis of v must be used in conjunction with the axis of d, and the right-hand axis of v must be used with the axis of H and the mid-vertical.

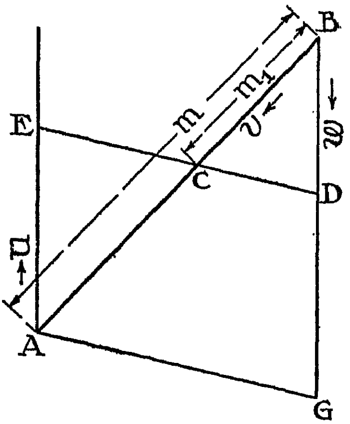


FIG. 46.

A useful chart, of character similar to Fig. 17, may be constructed to give powers and roots directly, and also logarithms to any base.

In Fig. 46 let AE be the axis of u, BA the axis of v, and BG the axis of w: also let the length of AB be m.

Let AE represent $\log u_1$ to a scale of $1'' = l_1$, and let BD represent $\log w_1$ to a scale of $1'' = l_2$.

Then, if AG be drawn parallel to ED, the triangles BAG and BCD are similar, and hence

$$\frac{BG}{BA} = \frac{BD}{BC}$$

i. e.

$$\frac{BD + EA}{m} = \frac{BD}{m_1}$$

$$\frac{\frac{\log w_1}{l_2} + \frac{\log u_1}{l_1}}{m} = \frac{\frac{\log w_1}{l_2}}{m_1}$$

$$\frac{l_1 \log w_1 + l_2 \log u_1}{l_1 l_2 m} = \frac{\log w_1}{l_2 m_1}$$

$$m_1 l_1 \log w_1 + m_1 l_2 \log u_1 = m l_1 \log w_1$$

or $$m_1 l_2 \log u_1 = l_1 \log w_1 (m - m_1)$$

$$\frac{m_1 l_2}{l_1(m - m_1)} \log u_1 = \log w_1$$

Now if $$v_1 = \frac{m_1 l_2}{l_1(m - m_1)}$$

$$v_1 \log u_1 = \log w_1$$

or, in general, $$v \log u = \log w$$

whence $$u^v = w.$$

To graduate the axis of v the various values of m_1 must be calculated.

$$v_1 = \frac{m_1 l_2}{l_1(m - m_1)}$$

$$v_1 l_1 m - v_1 l_1 m_1 = m_1 l_2$$

$$m_1(l_2 + v_1 l_1) = v_1 l_1 m$$

and $$m_1 = \frac{m v_1 l_1}{l_2 + v_1 l_1} \quad \text{or} \quad \frac{m v_1}{\frac{l_2}{l_1} + v_1}$$

Chart for Powers and Roots

Example 12.—Construct a chart to illustrate the equation $u^v = w$, the values of u varying from 1 to 10, and those of w varying from 1 to 1000. Show how this chart may be used to give logs of numbers to any base.

In Fig. 47 the axes of u and w are drawn 3¼″ apart and the line AB is made exactly 6″ long. The values of $\log u$ are set out using the scale-value $l_1 = \cdot 2$, and for the axis of w the scale-value l_2 is taken as ·5. Thus $m = 6$, $l_1 = \cdot 2$, $l_2 = \cdot 5$ and $\frac{l_2}{l_1} = 2\cdot 5$, so that $m_1 = \frac{6 v_1}{2\cdot 5 + v_1}$. For $v_1 = 2$, for example, $m_1 = \frac{6 \times 2}{2\cdot 5 + 2} = 2\cdot 67$ inches. Some points on the axis of v can be marked by joining suitable values of u and w. Thus $u = 10$ joined in turn to $w = 10$, 100 and 1000 gives the positions of the markings for $v = 1$, 2, and 3 respectively. Other points can be found by calculation, a few values being indicated in the table :—

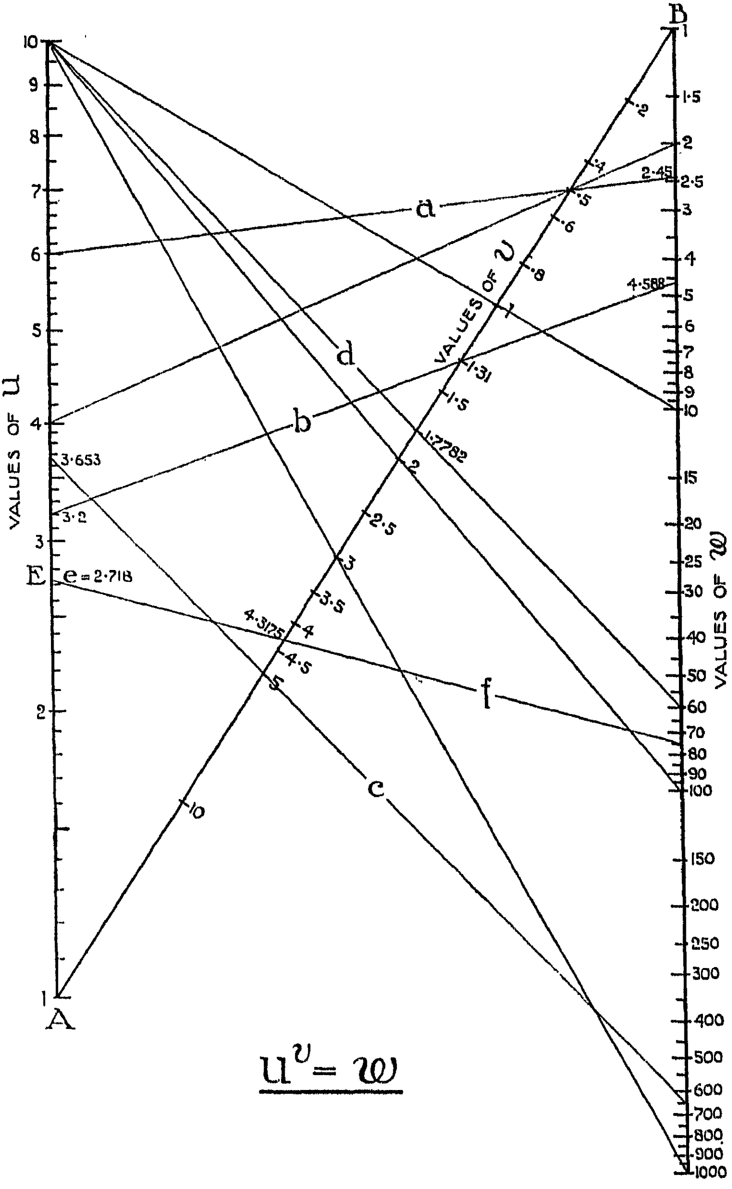


Fig. 47.—Chart for Powers and Roots.

v . .	·2	·4	·6	·8	1·5	2·5	3·5	4	10
m_1 .	·444	·826	1·162	1·455	2·25	3	3·5	3·69	4·8

To Use the Chart.—(*a*) To find the square root of 6. Here $u = 6$ and $v = \cdot 5$, hence join $u = 6$ to $v = \cdot 5$ and produce the line to cut the axis of w at 2·45. Then $\sqrt{6} = 2{\cdot}45$.

(*b*) To find the value of $3{\cdot}2^{1\cdot 31}$. Join $u = 3{\cdot}2$ to $v = 1{\cdot}31$ and produce the line to cut the axis of w at 4·588.

(*c*) To find the fifth root of 650. Here $u^5 = 650$, and hence it is necessary to join $w = 650$ to $v = 5$ and produce the line to cut the axis of u at 3·653. Thus $\sqrt[5]{650} = 3{\cdot}653$.

If $u^v = w$ then $\log_u w = v$ so that if the axis of w is taken as the scale of numbers, the axis of u gives the value of the base of the logarithm, and the logarithm is read off on the axis of v. For ordinary working the two bases needed are 10, for common logs, and e, *i. e.* 2·718, for natural or Napierian logs. The point E on the axis of u, at the value 2·718, is distant $\frac{1}{\cdot 2}(\log 2{\cdot}718 - \log 1)$, *i. e.* $5 \times \cdot 4343$ or 2·172 inches from A. Then common logs are found by drawing a series of lines radiating from $u = 10$, and for natural logs the lines are drawn from the point E.

Thus (*d*) To find the common log of 60. Join $u = 10$ to $w = 60$ and note the point of intersection, 1·7782, with the axis of v: then $\log_{10} 60 = 1{\cdot}7782$.

(*f*) To find the natural log of 75. Join E to 75 on the axis of w to intersect the axis of v at 4·3175. Then $\log_e 75 = 4{\cdot}3175$.

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MATHEMATICAL TABLES

TABLE I.—Trigonometrical Ratios

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
De-grees.	Radians.								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	Degrees
								Angle	

TABLE II.—LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

TABLE II. (*contd.*)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9														
95	9777	9782	9786	9791															
96	9823	9827	9832	983															
97	9868	9872	9877	988															
98	9912	9917	9921	992															
99	9956	9961	9965	996															

TABLE III.—ANTILOGARITHMS

CHECKED

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE III. (*contd*).

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20